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Four factor factorial design on residual curvatures of metal round bars in cross-roll straightening process

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ABSTRACT

Straight round bars are widely used as raw materials in numerous industrial applications, yet they are frequently delivered in a bent condition, making straightening a necessary preparatory step to ensure consistent product quality. Achieving acceptable levels of residual curvature after straightening is essential, as excessive deviation can affect subsequent machining and assembly processes. This study explores the use of statistical Factorial Design to support decision-making and establish reliable quality criteria related to residual curvature. The applicability of Three-Factor and Four-Factor Factorial Designs is assessed to determine the influence of key process parameters and their interactions on straightening performance. The results provide insights that can help improve process optimization and the selection of bar lots that meet required curvature tolerances.

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1. INTRODUCTION

Round Bars produced in re-rolling industries are often not perfectly straight, in short straightness is compromised in general after production of round bars. Degree of straightness in commercially available round bars varies from 1 in 750 to 1 in 5000 [1]. To improve straightness, it is necessary that round bars are passed through straightening machines, preferably Two Cross-Roll straighteners. The cross-rolls are generally kept at an angle called helix angle. The helix angle helps in rotating the round bar and simultaneously moves the bar forward. It has already been established that a combination of Roller Diameter and Helix Angle is significantly important for desired output i.e. reduced curvature and speed of bar's forward motion. Several factors are involved in the process of bar straightening. Beside roller diameter, helix angle there are other factors such as roller speed, bending amount, bar diameter and modulus of elasticity etc. The statistical analysis using factorial design and analysis of variance with several factors like roller diameter, helix angle, bar diameter, modulus of elasticity of bar etc. plays an important role for proper straightening.

In this research work, Four-Factor Factorial Design has been implemented considering roller diameter, helix angle, bar diameter and modulus of elasticity as influencing factors. Utility of four-factor in bar straightening now focusses on factors of machine side and material side together. During the process of bar straightening, factors relating to bar straightening machine and working materials are actually involved at the same time, hence it is quite pertinent to consider factors of both sides simultaneously. In this paper four-factor factorial design associated with two cross roll straightening process has been thoroughly investigated by using analysis of variance.

2. LITERATURE REVIEW

A brief literature review reveals that extensive research is going on in bar straightening in different areas including statistical considerations. Several researchers have worked in the area of bar straightening over a long span of time, earliest being the work of Haruo Tokunaga (1961) on Roller Straightener [2]. By the year 1974, there have been tremendous advancement which was highlighted by G.E. Kemshall. Theoretical aspects of bar straightening initially started by Yu & Johnson [3] and later especially the mechanics of bar straightening was developed by Das Talukder et al. in the year 1981 [4],[5]. Li et al. (1999) indicated clearly that irrespective of accuracy of industrial process in the production of steel tubes or bars, straightening process is almost essential. This is required

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in order to correct out-of-straightness and out-of-roundness [6]. In the year 2001, Marcus Paech focussed on wire straightening and included material characteristics like modulus of elasticity, yield point, sectional geometry, initial curvature and helix angle [7]. In the year 2008, Marcus Paech has again shown that intelligent and flexible machinery technology, excellent process planning and optimisation are essential for production process design thus intending to remove or modify curvature in the process materials [8]. Kato et al. (2014) stated that in order to improve quality of round bars and productivity, researchers have looked into various aspects of cross-roll arrangements [9]. Roll gap, rotational speed of top and bottom rolls and skew angles happen to play significant role. Several researchers worked on modelling, numerical modelling and simulation area of bar straightening. In the year 1989, Dvorkin & Medina developed Finite Element model to analyse straightening and rounding of steel tubes [10]. Mischke and Jonca worked on simulation of the roller straightening process in the year 1992 and stated that roller straightening itself is a pretty complicated process thus specifically mentioning that there are three different periods of state viz. the entry
period of material, period of stabilised process condition and thirdly the period of exit after straightening [11]. They highlighted that roller diameter is a factor among many factors that affect straightening process. In the year 1996, Macura & Petruska used finite element system in ANSYS software by 3D simplified numerical model for the purpose of numerical analysis [12]. Wu et al. (2000) also used mathematical model on precision straightened bar involving iterative function in the treatment [13]. In the year 2008, Mutrux et al. have studied about cross-roll straightening process in a simulated situation using LS-Dyna [14]. Use of FEM was a preferred area of many researchers in bar straightening. In the year 2010, Tian et al. [15] and Song et al. [16] used FEM in their work. In 2012, Yali & Herong stated that the process guarantees straightening accuracy of the bar while residual curvature of the same bar reaches equivalent curvature [17]. In the year 2013, Mishra et al. made an attempt to consider statistical approach on the roundness of cylindrical bar by the application of Taguchi method. Mishra et al. worked on Experimental Design and worked on mean values of roundness and used S/N ratio in L9 Orthogonal array.
Earlier, Roy & Pal worked on Two-Factor Factorial Design in the year 2022 [18] which formed basis of this publication of considering two more factors other than roller diameter (D) and helix angle (α) i.e. bar diameter (B) and modulus of elasticity (E) in the bar straightening process. In short, statistical considerations is gaining importance in the field of bar straightening.
In a recent publication, Ren et al. worked in straightening analysis and parameter optimisation of rotary hub steel straightening machine using Three-Factor namely roller tilt angle, bending moment and hub speed on the straightness of steel bar [19].
Although there is considerable amount of reported works in cross roll bar straightening process, statistical analysis considering several influencing factors associated with straightening process are not well reported till date to the best of knowledge of authors.

3. THEORETICAL ASPECTS OF BAR STRAIGHTENING

Theoretical aspects have been discussed in detail in several research papers including Roy & Pal (2022) [18]. Motion of round bar in cross-roll straightening machine has been briefly described in the photograph and schematic diagram shown in Fig.1(a) and (b) respectively where two cross-rolls are at an angle α called helix angle with respect to axial motion of round bar and throughput velocity, v_x.
If cross-rolls are rotating about their axes at angular speed ω and radius of each roller is R, then both throughput velocity and tangential velocity v_t can be expressed as below:

$$v_x = \omega R \sin\alpha \quad \text{and} \tag{1}$$

$$v_t = \omega R \cos\alpha \tag{2}$$

Fig. 1a Convex and Concave Rollers showing round bar in between during the straightening process.

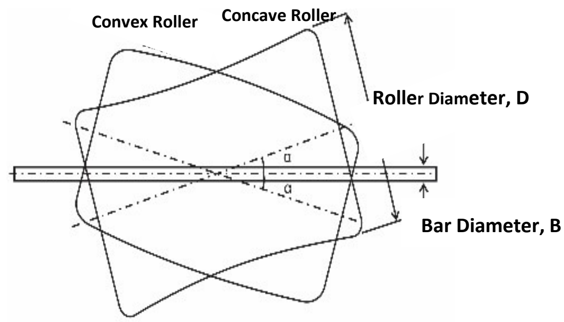


Fig. 1b Convex Roller and Concave Roller making an angle α (Helix Angle) with Round Bar

Figure 2 shows the entire straightening machine (Model: SMH-25, Make: Bhambra Engineering Works) where the round bar has been deployed for straightening in Two Cross-Roll arrangements.

Fig. 2 Straightening Process of round bar in Two Cross-Roll straightening machine (SMH-25).

3.1. Assumptions Made During Bar Straightening Analysis

Certain assumptions have been made during the analysis of bar straightening process which are as below:

(i) Work hardening is elastic region is present in the straightened bar.

(ii) Cross-sectional area of round bar is circular in pure bending condition and neutral axis passes through the centre

(iii) Bars outside shapes are round.

(iv) Strain variation in bar section at any point is proportional to the distance from perpendicular point on neutral axis.

(v) Material is perfectly elastic-plastic, yield point and its stress-strain relations is same in uniaxial tension and compression.

(vi) Roller Diameters are same dimensionally.

In case first assumption is not fully satisfying which means that hardening is not present in the straightened bar. In that case the deformation results will be affected due to reduced yield strength and increased deformation. The bar will exhibit increased curvature which may lead to larger deformations. The material may exhibit the Bauschinger effect showing lower yield strength in reverse direction than in the original direction. The implication could be lower load-carrying capacity and increased failure risk.

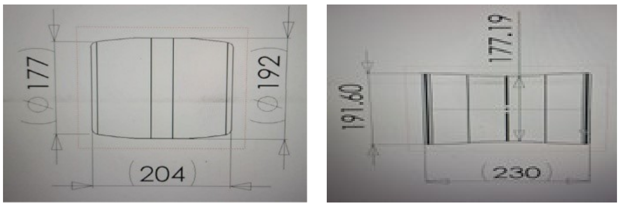
For the second assumption if not fully satisfying which means that cross-sectional area of round bar is not circular in pure bending and neutral axis does not pass through the centre then there will be non-uniform distribution of stresses across the sections leading to unexpected deformation patterns.

If third assumption is not fully satisfying which means outside shape is not round, then deformation results during straightening will vary in several ways. Non-uniform deformations will result into variations in curvature along the bar length leading to difficulty in achieving uniform straightening.

If fourth assumption is not fully satisfying which implies that the beam is not behaving according to Euler-Bernoulli beam theory and will cause non-proportional strain variation leading to non-linear strain distribution and complex stress distribution.

For the fifth assumption, if not fully satisfying, then there is a possibility on non-linear stress-strain curve thus material may show non-linear elastic behaviour. Strain curve will not be a straight line. The material may show variable stiffness which will depend on strain level affecting deformation results under bending and reverse bending with implication of possibility of complex deformation behaviour thus possibly leading to complex deformation patterns.

The sixth assumption being the roller diameters are of same dimension and in reality, the roller diameters are insignificantly different. Fig.3(a)&3(b) show the dimensional aspect (in mm) of straightening machine Model: SMH-25. The diametrical dimensions are within machining tolerances, hence can be assumed practically of same dimension.



(a) (b)

Fig. 3 Diagram of (a) convex roller and (b) concave roller with dimensions in mm as in straightening machine model: smh25

4. Analysis of Residual Curvature in The Process of Bar Straightening

Residual curvatures are nearly inevitable in most round bars during processing at rolling mills at production stage and thereafter during transportation and material handling. Therefore, there have been emergence of bar straightening process primarily to reduce existing residual curvatures. Moment-curvature relationship is considered most important governing equation in bar straightening. The moment-curvature relationship in non-dimensional form is given below:

$$\bar{M} = \bar{\kappa} \qquad 0 \le \bar{M} \le 1 \qquad (3)$$

$$\bar{M} = \bar{M}(\bar{\kappa}) \qquad 1 \le \bar{M} \qquad (4)$$

where, $\bar{M} = M/M_y$ and $\bar{\kappa} = \kappa/\kappa_y$.

M_y is the yield moment at yield curvature κ_y for a section considered [20]. $\bar{M}(\bar{\kappa})$ is the function $\bar{\kappa}$ depending on the stress-strain relationship of the material beyond yield point [4].

5. FOUR-FACTOR FACTORIAL DESIGN OF RESIDUAL CURVATURES IN BAR STRAIGHTENING

Many experiments involve study of effects of factors which may be two or more. In general, factorial designs are said to be most efficient for this type of experiments. The observations in a factorial experiment can be described by a model. There are several ways to write the model for a factorial experiment. The effects model is described here for residual curvature, κ_{ijkl} for a four-factor factorial design and analysis of variance model of final residual curvatures in bar straightening process is given below:

$$\kappa_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + \delta_l + (\tau\beta)_{ij} + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\delta)_{il} + (\beta\delta)_{jl} + (\gamma\delta)_{kl} + (\tau\beta\gamma)_{ijk} + (\tau\beta\delta)_{ijl} + (\tau\gamma\delta)_{ikl} + (\beta\gamma\delta)_{jkl} + (\tau\beta\gamma\delta)_{ijkl} + \epsilon_{ijklm}$$

$$\begin{cases} i = 1,2,\dots,a \\ j = 1,2,\dots,b \\ k = 1,2,\dots,c \\ l = 1,2,\dots,d \\ m = 1,2,\dots,n \end{cases} \quad (5)$$

where,

μ is the overall mean effect of the roller diameter, helix angle, bar diameter and modulus of elasticity
τ_i is the effect of the i^{th} level of Roller Diameter (D) factor
β_j is the effect of the j^{th} level of Helix Angle (α) factor
γ_k is the effect of the k^{th} level of Bar Diameter (B) factor
δ_l is the effect of the l^{th} level of Modulus of Elasticity (E) factor
$(\tau\beta)_{ij}$ is the effect of interaction between τ_i and β_j
$(\tau\gamma)_{ik}$ is the effect of interaction between τ_i and γ_k
$(\beta\gamma)_{jk}$ is the effect of interaction between β_j and γ_k
$(\tau\delta)_{il}$ is the effect of interaction between β_j and γ_k
$(\beta\delta)_{jl}$ is the effect of interaction between β_j and δ_l
$(\gamma\delta)_{kl}$ is the effect of interaction between γ_l and δ_l
$(\tau\beta\gamma)_{ijk}$ is the effect of interactions among τ_i, β_j and γ_k
$(\tau\beta\delta)_{ijl}$ is the effect of interactions among τ_i, β_j and δ_l
$(\tau\gamma\delta)_{ikl}$ is the effect of interactions among τ_i, γ_k and δ_l
$(\beta\gamma\delta)_{jkl}$ is the effect of interactions among β_j, γ_k and δ_l
$(\tau\beta\gamma\delta)_{ijkl}$ is the effect of interactions among τ_i, β_j, γ_k and δ_l
ϵ_{ijkl} is the random error component.

All the three factors are assumed to be fixed and the treatment effects are defined as deviations from overall mean so that

for Roller Diameter (D), $\sum_i^a \tau_i = 0$ (6)

for Helix Angle (α), $\sum_j^b \beta_j = 0$ (7)

for Bar Diameter (B), $\sum_k^c \gamma_k = 0$ and (8)

for Modulus of Elasticity (E), $\sum_l^d d_l=0$ (9)

Similarly, the interaction effects among roller diameter (D), helix angle (α), bar diameter (B) and modulus of elasticity (E) are fixed and defined in following way:

$$\sum_i^a (\tau\beta)_{ij} = \sum_j^b (\tau\gamma)_{ik} = \sum_k^c (\beta\gamma)_{jk} = \sum_m^d (\tau\delta)_{il} = \sum_m^d (\beta\delta)_{jl} = \sum_l^d (\gamma\delta)_{kl} = 0 \quad (10)$$

Since there are *n* replicates of the experiment, there are *abcdn* observations.

This is a case of four-variable factorial design, two variables are at machine side while other two variables are on material side. Considering all variables are of equal interest, testing hypothesis about the equality of Roller Diameter treatment effects, it can be written as

Null Hypothesis H_o: $\tau_1 = \tau_2 = \tau_3 \dots\dots = \tau_a$ (11)

Alternate Hypothesis H_1: at least one $\tau_i \neq 0$ (12)

For the equality of helix angle treatment effects, it can be written as

H_o: $\beta_1 = \beta_2 = \beta_3 \dots\dots = \beta_b$ (13)

H_1: at least one $\beta_j \neq 0$ (14)

For the equality of bar diameter treatment effects, it can be written as

H_o: $\gamma_1 = \gamma_2 = \gamma_3 \dots\dots = \gamma_c$ (15)

H_1: at least one $\gamma_k \neq 0$ (16)

Similarly, For the equality of modulus of elasticity treatment effects, it can be written as

H_o: $\delta_1 = \delta_2 = \delta_3 \dots\dots = \delta_d$ (17)

H_1: at least one $\delta_1 \neq 0$ (18)

6. STATISTICAL ANALYSIS OF THE ROLLER DIAMETER, HELIX ANGLE, BAR DIAMETER AND MODULUS OF ELASTICITY FACTORS IN BAR STRAIGHTENING PROCESS

Statistical analysis leads to logical conclusion in decision making process and helps qualitatively to accept a test hypothesis. Analysis of variance table through F-Test gives statistical inference when compared with critical values of F Distribution table. In the present case intended variable is residual curvature, κ . Various levels based on factors of residual curvature is presented below.

Let $\kappa_{i....}$ denote the total of all observations under the i^{th} level of Roller Diameter factor (D),
$\kappa_{.j...}$ denote the total of all observations under the j^{th} level of Helix Angle factor (α),
$\kappa_{..k..}$ denote the total of all observations under the k^{th} level of Bar Diameter factor (B),
$\kappa_{...l.}$ denote the total of all observations under l^{th} level of Elastic Modulus factor (E),
$\kappa_{.....}$ denotes grand total of all observations.

Assuming that D, α , d and E are fixed, the analysis of variance table is shown in Table 2.

The *F* tests on main effects and interactions follow directly from the expected mean squares. Usually, the ANOVA computations would be done using statistical software package. However, occasionally manual computing formulas for the sums of squares as in Table 1 is useful. The total sum of squares is found in the usual way as

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{c} \sum_{l=1}^{d} \sum_{m=1}^{n} \kappa_{ijklm}^2 - \frac{\kappa_{.....}^2}{abcdn} \quad (19)$$

The sum of squares for the main effects is found from the totals for factors D($\kappa_{i....}$), α($\kappa_{.j..}$), B($\kappa_{..k.}$) and E($\kappa_{...l}$) as follows:

$$SS_D = \frac{1}{bcdn} \sum_{i=1}^{a} \kappa_{i....}^2 - \frac{\kappa_{.....}^2}{abcdn} \quad (20)$$

$$SS_\alpha = \frac{1}{acdn} \sum_{j=1}^{b} \kappa_{.j...}^2 - \frac{\kappa_{.....}^2}{abcdn} \quad (21)$$

$$SS_B = \frac{1}{abdn} \sum_{k=1}^{c} \kappa_{..k..}^2 - \frac{\kappa_{.....}^2}{abcdn} \quad (22)$$

$$SS_E = \frac{1}{abcn} \sum_{l=1}^{d} \kappa_{...l.}^2 - \frac{\kappa_{.....}^2}{abcdn} \quad (23)$$

Computations can be done for the interaction sums of squares. It is frequently helpful to collapse the original data table into two-way tables to compute these quantities. The sums of squares are found from Eq.(24) to Eq.(38).

$$SS_{D\alpha} = \frac{1}{cn} \sum_{i=1}^{a} \sum_{j=1}^{b} \kappa_{ij..}^2 - \frac{\kappa_{.....}^2}{abcdn} - SS_D - SS_\alpha \quad (24)$$

$$= SS_{Subtotals(D\alpha)} - SS_D - SS_\alpha \quad (25)$$

$$SS_{DB} = \frac{1}{bn} \sum_{i=1}^{a} \sum_{k=1}^{c} \kappa_{i.l.}^2 - \frac{\kappa_{.....}^2}{abcdn} - SS_D - SS_B \quad (26)$$

$$= SS_{Subtotals(DB)} - SS_D - SS_B \quad (27)$$

$$SS_{\alpha B} = \frac{1}{an} \sum_{j=1}^{b} \sum_{k=1}^{c} \kappa_{.jk.}^2 - \frac{\kappa_{.....}^2}{abcdn} - SS_\alpha - SS_B \quad (28)$$

$$= SS_{Subtotals(\alpha B)} - SS_\alpha - SS_B \quad (29)$$

$$SS_{D\alpha B} = \frac{1}{n} \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{c} \kappa_{ijk.}^2 - \frac{\kappa_{.....}^2}{abcdn} - SS_D - SS_\alpha - SS_B - SS_{D\alpha} - SS_{DB} - SS_{\alpha B} \quad (30)$$

$$= SS_{Subtotals(D\alpha B)} - SS_D - SS_\alpha - SS_B - SS_{D\alpha} - SS_{DB} - SS_{\alpha B} \quad (31)$$

$$SS_{D\alpha E} = \frac{1}{n} \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{l=1}^{d} \kappa_{ijl.}^2 - \frac{\kappa_{.....}^2}{abcdn} - SS_D - SS_\alpha - SS_B - SS_{D\alpha} - SS_{DB} - SS_{\alpha B} \quad (32)$$

$$= SS_{Subtotals(D\alpha B)} - SS_D - SS_\alpha - SS_B - SS_{D\alpha} - SS_{DB} - SS_{\alpha B} \quad (33)$$

$$SS_{DBE} = \frac{1}{n} \sum_{i=1}^{a} \sum_{k=1}^{c} \sum_{l=1}^{d} \kappa_{ikl.}^2 - \frac{\kappa_{.....}^2}{abcdn} - SS_D - SS_B - SS_E - SS_{DB} - SS_{DE} - SS_{BE} \quad (34)$$

$$= SS_{Subtotals(DBE)} - SS_D - SS_B - SS_E - SS_{DB} - SS_{DE} - SS_{BE} \quad (35)$$

$$SS_{\alpha BE} = \frac{1}{n} \sum_{j=1}^{b} \sum_{k=1}^{c} \sum_{l=1}^{d} \kappa_{jkl.}^2 - \frac{\kappa_{.....}^2}{abcdn} - SS_D - SS_B - SS_E - SS_{DB} - SS_{DE} - SS_{BE} \quad (36)$$

$$= SS_{Subtotals(DBE)} - SS_D - SS_B - SS_E - SS_{DB} - SS_{DE} - SS_{BE} \quad (37)$$

The error sum of squares may be found by subtracting the sum of squares for each main effect and the interaction from the total sum of squares or by

$$SS_\epsilon = SS_T - SS_{Subtotals(DBE)} \quad (38)$$

Mean squares of the factors in four-factor factorials and treatments are as below:

$$MS_D = \frac{SS_D}{a-1} \quad (39)$$

$$MS_\alpha = \frac{SS_\alpha}{b-1} \quad (40)$$

$$MS_B = \frac{SS_\alpha}{c-1} \quad (41)$$

$$MS_E = \frac{SS_\alpha}{d-1} \quad (42)$$

$$MS_{D\alpha} = \frac{SS_{D\alpha}}{(a-1)(b-1)} \quad (43)$$

$$MS_{DB} = \frac{SS_{DB}}{(a-1)(c-1)} \quad (44)$$

$$MS_{\alpha\beta} = \frac{SS_{\alpha\beta}}{(b-1)(c-1)} \quad (45)$$

$$MS_{DE} = \frac{SS_{DE}}{(a-1)(d-1)} \quad (46)$$

$$MS_{\alpha E} = \frac{SS_{\alpha E}}{(b-1)(d-1)} \quad (47)$$

$$MS_{BE} = \frac{SS_{BE}}{(c-1)(d-1)} \quad (48)$$

$$MS_{D\alpha\beta} = \frac{SS_{D\alpha\beta}}{(a-1)(b-1)(c-1)} \quad (49)$$

$$MS_{D\alpha E} = \frac{SS_{D\alpha E}}{(a-1)(b-1)(d-1)} \quad (50)$$

$$MS_{DBE} = \frac{SS_{DBE}}{(a-1)(c-1)(d-1)} \quad (51)$$

$$MS_{\alpha BE} = \frac{SS_{\alpha BE}}{(b-1)(c-1)(d-1)} \quad (52)$$

$$MS_{D\alpha BE} = \frac{SS_{D\alpha BE}}{(a-1)(b-1)(c-1)(d-1)} \quad (53)$$

$$MS_\epsilon = \frac{SS_\epsilon}{abcd(n-1)} \quad (54)$$

7. ANALYSIS OF VARIANCE FOR FINAL RESIDUAL CURVATURE ON FOUR-FACTOR FACTORIAL DESIGN

The final residual curvature was evaluated through an ANOVA procedure considering four primary variables of the cross-roll bar straightening process: roller diameter (D), helix angle (α), bar diameter (B), and modulus of elasticity (E). The analysis examines not only the individual effects of these factors but also their contribution through two-factor, three-factor, and four-factor interaction terms. Accordingly, the ANOVA framework incorporates the interaction components Dα, DB, αB, DE, αE, BE, DαB, DαE, DBE, αBE, and the four-factor interaction DαBE, together with the associated error term (ϵ). A detailed summary of all included factors, interactions, and their statistical significance is presented in Table 1.

Table 1 – *The Analysis of Variance Table for the Four-Factor Fixed Effects Model*

Source of variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Roller Diameter(D)	SS_D	a-1	MS_D	$F_0=\frac{MS_D}{MS_\epsilon}$
Helix Angle (α)	SS_α	b-1	MS_α	$F_0=\frac{MS_\alpha}{MS_\epsilon}$
Bar Diameter (B)	SS_d	c-1	MS_B	$F_0=\frac{MS_B}{MS_\epsilon}$
Modulus of Elasticity(E)	SS_E	d-1	MS_E	$F_0=\frac{MS_E}{MS_\epsilon}$
Dα	$SS_{D\alpha}$	(a-1)(b-1)	$MS_{D\alpha}$	$F_0=\frac{MS_{D\alpha}}{MS_\epsilon}$
DB	SS_{DB}	(a-1)(c-1)	MS_{DB}	$F_0=\frac{MS_{DB}}{MS_E}$
αB	$SS_{\alpha B}$	(b-1)(c-1)	$MS_{\alpha B}$	$F_0=\frac{MS_{\alpha B}}{MS_\epsilon}$
DE	SS_{DE}	(a-1)(d-1)	MS_{DE}	$F_0=\frac{MS_{DE}}{MS_\epsilon}$
αE	$SS_{\alpha E}$	(b-1)(d-1)	$MS_{\alpha E}$	$F_0=\frac{MS_{\alpha E}}{MS_\epsilon}$
BE	SS_{BE}	(c-1)(d-1)	MS_{BE}	$F_0=\frac{MS_{BE}}{MS_\epsilon}$
DαB	$SS_{D\alpha B}$	(a-1)(b-1)(c-1)	$MS_{D\alpha B}$	$F_0=\frac{MS_{D\alpha B}}{MS_\epsilon}$
DαE	$SS_{D\alpha E}$	(a-1)(b-1)(d-1)	$MS_{D\alpha E}$	$F_0=\frac{MS_{D\alpha E}}{MS_\epsilon}$
DBE	SS_{DBE}	(a-1)(c-1)(d-1)	MS_{DBE}	$F_0=\frac{MS_{DBE}}{MS_\epsilon}$
αBE	$SS_{\alpha BE}$	(b-1)(c-1)(d-1)	$MS_{\alpha BE}$	$F_0=\frac{MS_{\alpha BE}}{MS_\epsilon}$
DαBE	$SS_{D\alpha BE}$	(a-1)(b-1)(c-1)(d-1)	$MS_{D\alpha BE}$	$F_0=\frac{MS_{D\alpha BE}}{MS_\epsilon}$
Error	SS_ϵ	$abcd(n$-1)	MS_ϵ	
Total	SS_T	$abcdn$-1		

8. DISCUSSION

The theoretical aspects of the bar straightening process have been examined through a comprehensive Three-Factor Factorial analysis, considering three key process variables: roller diameter, helix angle, and bar diameter. These factors fundamentally influence the mechanics of straightening, which is clearly reflected in the derived equations describing curvature reduction and deformation behavior. Building upon this foundation, the study further extends the analysis by incorporating the modulus of elasticity as a fourth factor, thereby developing a Four-Factor Factorial Design aimed at predicting and optimizing residual curvatures.

This four-factor framework enables a more holistic statistical representation of the straightening process by integrating both machine-related (roller diameter and helix angle) and material-related variables (bar diameter and modulus of elasticity). The detailed statistical treatment includes the computation of sums of squares, mean squares, and the allocation of degrees of freedom corresponding to the respective levels of each factor. Such an approach provides deeper insight into the relative influence and interaction effects of the factors on final bar curvature.

The expanded model demonstrates that variations in material properties—particularly modulus of elasticity—play a significant role in residual deformation, a dimension often overlooked in earlier studies focused predominantly on machine parameters. Consequently, the present analysis not only broadens the applicability of factorial design in bar straightening research but also highlights the importance of integrating mechanical and material characteristics within a unified statistical framework.

Moreover, this study lays the groundwork for future investigations by offering a scalable experimental design structure. Additional variables—such as roller spacing, feed rate, surface finish, or thermal effects—may be incorporated into extended factorial or response-surface-based approaches to achieve even more accurate predictions of straightening performance. The insights gained here may ultimately contribute to improved process control, enhanced bar quality, and the development of more advanced straightening equipment.

In summary, by bridging machine and material perspectives through a Four-Factor Factorial framework, the present work provides a more comprehensive understanding of the bar straightening process. This integrated approach represents a meaningful step toward the systematic application of statistical methodologies in the modeling, optimization, and improvement of industrial straightening operations.

9. CONCLUSION

The application of ANOVA based on the selected factors—Roller Diameter, Helix Angle, Bar Diameter, and Modulus of Elasticity—has been conducted theoretically. Using the derived equations, an F-test can be performed, and the corresponding F-values from standard statistical tables may be compared with actual experimental observations to draw statistically valid conclusions regarding the proposed hypotheses. This approach serves as a useful tool for assessing the individual and combined roles of these factors in the bar straightening process, thereby supporting quality improvement efforts.

This study provides two key contributions that highlight its novelty. First, the statistical analysis incorporates both machine-related parameters and material-related parameters, offering a more holistic evaluation of the straightening process. Second, a four-factor factorial design has been developed, integrating roller diameter and helix angle as machine factors, and bar diameter and modulus of elasticity as material factors, to better understand and reduce residual curvature along bar segments.

The findings also open pathways for future research to investigate additional machine- and material-based factors within similar statistical frameworks. Considering the results and the potential for further expansion, the proposed four-factor factorial design constitutes a meaningful contribution to the evolving application of statistics in bar straightening process analysis.

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