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Evaluation of the algorithms for recovering reflectance from virtual digital camera response

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Abstract:
In the recent years many new methods for quality control in graphic industry are proposed. All of these methods have one in common – using digital camera as a capturing device and appropriate image processing method/algorithm to obtain desired information. With the development of new, more accurate sensors, digital cameras became even more dominant and the use of cameras as measuring device became more emphasized. The idea of using camera as spectrophotometer is interesting because this kind of measurement would be more economical, faster, widely available and it would provide a possibility of multiple colour capture with a single shot. This can be very useful for capturing colour targets for characterization of different properties of a print device. A lot of effort is put into enabling commercial colour CCD cameras (3 acquisition channels) to obtain enough of the information for reflectance recovery. Unfortunately, RGB camera was not made with the idea of performing colour measurements but rather for producing an image that is visually pleasant for the observer. This somewhat complicates the task and seeks for a development of different algorithms that will estimate the reflectance information from the available RGB camera responses with minimal possible error. In this paper three different reflectance estimation algorithms are evaluated (Orthogonal projection,Wiener and optimized Wiener estimation), together with the method for reflectance approximation based on principal component analysis (PCA). The aim was to perform reflectance estimation pixel wise and analyze the performance of some reflectance estimation algorithms locally, at some specific pixels in the image, and globally, on the whole image. Performances of each algorithm were evaluated visually and numerically by obtaining pixel wise colour difference and pixel wise difference of estimated reflectance to the original values. It was concluded that Wiener method gives the best reflectance estimation while produces the best colour fit as well.

Keywords: reflectance recovery algorithms, pixel wise estimation, PCA, Wiener estimation

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Introduction

Obtaining colour information in graphic industry implies using colour measurement devices such as colourimeters and spectrophotometers. These devices are vide spread and can give fast, consistent and reliable values if areas of solid colour are measured. Since measurement is performed point-wise and the area to be evaluated is restricted to the device aperture, it is impossible to measure spatial variations or to obtain any additional image information. Digital cameras, on the other hand, can capture broader area of the coloured surfaces and can record colour information of a sample with additional information about texture and other surface properties. With this their application can be extended from imaging to measuring device.

Two of the relatively recent applications of the imaging systems based on cameras are estimating the XYZ tristimulus values and reflectance spectrum of a colour sample from its corresponding system's response digital levels. Reflectance is an illumination independent property of an object. When captured by digital camera it is captured in a form of a colour signal that is illuminant dependent. Therefore, the final camera response is dependent on the illumination used for the capture. Many applications that are focused on object analysis request pure reflectance information; hence it is very useful to find a way to get this information from the RGB sensor response values.

A perfect case for obtaining reflectance information would be to use a hyperspectral camera and obtain a spectral image that will contain reflectance information for each pixel of the given scene. A spectral image is an image which has both spatial and spectral resolution and on the spectral axis it has tens of components, whereas a conventional RGB image has only three. What makes the spectral image more accurate than RGB image is the fact that the visible range (380-780 nm) is sampled with more suitable wavelength interval giving a more precise reproduction (Mansouri et al, 2005). Unfortunately hyperspectral cameras are not easy, fast, cheap or widely available devices. Instead it is better, for example, to use an RGB camera and tries to estimate the reflectance based on some priory knowledge contained in the training data (Stigell et al, 2007; Mansouri et al, 2008).

Since the spectral sensitivities of most of the commercial colour CCD cameras do not verify the Luther condition i.e., are not linear transformation of the CIE matching functions (Rigueiro, 2009), it is necessary to perform a characterization process in order to define the correspondence between system response (RGB values) and desirable output (XYZ, CIELAB, reflectance spectrum). When reconstructing reflectance is a goal, the method of obtaining desired information from camera that is most widely used is based on capturing the set of standard samples and combining obtained camera's response with the values of the same samples obtained by spectrophotometric measurement (Sharma and Trussel, 1993; Harderberg, 2001; Cheung et al., 2004; Alsam and Lenz, 2007). The camera's response to the standard samples is related to the sample's reflectance spectra by the equation (Rigueiro, 2009):

$$Xp = P\,S\lambda \tag{1}$$

where Xp is a nx1 column vector whose components are the camera's responses to the n standard samples, $S\lambda$ is a mx1 column vector that represents the camera's spectral sensitivity, and P is a nxm matrix where each column corresponds to the energy reflected by each standard sample at m wavelengths, including the spectral power distribution of the illuminant, the reflectance spectrum of the standard sample, and the transmittance of the optical path between the sample and the camera. Some of the techniques used to solve the equation and estimate the $S\lambda$ vector are the Moore-Penrose pseudo-inverse, the Wiener estimation and the principal component analysis (PCA).

Slavuj (Slavuj, 2010) tested the possibility of using camera as a spectrophotometer for measuring test target for offset, ink-jet and electro-photographic prints on different substrates. He showed that the reconstruction error is a function of mean reflectance values within samples, where those with highest mean reflectance value have the largest reconstruction error. It implies that high gamut/dynamic range media caries a large variation in reflectance within colours and that PCA have no ability to adapt and that low reflective surfaces could be accurately reconstructed from the input basis vectors formed from high reflective surfaces (Slavuj, 2010).

Taking all this into account and due to the fact that the hyperspectral imaging is not widely available, the goal of this paper was to evaluate the possible mathematical solutions for reflectance estimation from the RGB camera responses with the idea to achieve fast and as accurate results as possible. In all the available reflectance estimation models the idea is to look for a function that will map camera response to the reflectance signal. The same idea was followed in this paper, where reflectance was estimated for each pixel in a test image. Three physical models were evaluated, together with PCA approximation from the defined set of training data. Overall, four recovery methods were tested:

1. Recovery by using first three eigenvectors of a training set (PCA)
2. Orthogonal projection

3. Wiener estimation
4. Optimized Wiener estimation

PCA method produces an approximation of the reflectance for the test image by using three eigenvectors corresponding to three largest eigenvalues. Using the set of Munsell colours, several authors reported high accuracy in reconstruction using different number of principal components. Eem, Shin (Eem et al, 1994) and Imai (Imai et al, 1996) showed that three components are sufficient for this task. As a first reflectance estimation we chose the approximation of the reflectance for the test image by using the simulated capture values and pixel wise orthogonal projection. The second method uses a pixel wise Wiener estimation whose purpose is to make estimations from low-dimensional data into high-dimensional data. It is shown that the Wiener method is quite simple to implement and at the same time it provides accurate estimates (Stigell et al, 2007; Mansouri et al, 2005). Finally the third approach introduces a parameter '*rho*' into the Wiener pixel wise estimation algorithm with the aim to optimize the performance of the classical Wiener algorithm. "Theoretical background" section provides more information about the chosen algorithms; "Method" covers the implementation more in depth, while the results are presented in the "Results and discussion" part of this paper.

Theoretical background

For more than half a century Principal Component Analysis (PCA) has been successfully used as an efficient mathematical tool for dimensionality reduction and recovery of spectral data. Cohen (Cohen, 1964) was the first contributing to the usage of this method by publishing the first three principal components of a subset of 150 out of 433 Munsell chips reflectance spectra. Later on, several researches used PCA method for extracting the suitable number of basis vectors that are able to define different spectral datasets (Eem et al., 1994; Fukunaga, 1990; Webb, 2002; Connah et al., 2001). For instance, Dannemiller (Dannemiller, 1992) investigated only natural objects and concluded that three eigenvectors are sufficient for representing their spectral reflectance.

The research in this field continues with Eem et al. (Eem et al., 1994) showing that the first three basis vectors of the reflectance spectra of 1,565 glossy Munsell colour chips accurately reconstruct the Macbeth Colour Checker spectral data. The list of the available research is very long coming down to the conclusion that the purpose of PCA is to identify the important features of a given dataset and reduce its dimensionality while preserving as much information about the variance as possible (Fukunaga, 1990; Webb, 2002). The basis vectors of a dataset are the eigenvectors calculated from its covariance matrix and they can be used to describe the given dataset. Connah and his coworkers (Connah et al., 2001) took this approach forward by applying the PCA method for obtaining basis vectors for spectral recovery from camera responses. The number of basic functions used in the recovery process was always equal to the number of sensor channels. This method has its foundation in the fact that the vast majority of reflectance spectra for natural and man-made surfaces are smooth functions of wavelength.

For reflectance estimation Wiener estimation method is widely used. Wiener filtering in general has many applications in the area of imaging science. For instance in digital image processing applications it is a widely used to reduce Gaussian noise, while in colour science can be used to estimate reflectance from camera response data on a pixel by pixel basis (Urban et al., 2008). In reflectance estimation task the goal is to reconstruct the spectrum which is a high dimensional signal (at least 81 values for a spectrum sampled in 5nm) from a low dimensional signal coming from the camera (usually 3 values: R, G, B). This makes this problem an ill-posed problem. Hence the purpose of the Wiener estimation is to make estimations from low-dimensional data into high dimensional data (Haneishi et al., 2000).

The Wiener estimation is one of the conventional estimation methods which is quite simple and provides accurate estimates. Because of its simplicity it was tested by many authors and many improvements had been proposed (Stigell et al, 2007; Haneishi et al, 2000). On the other hand Wiener algorithm cannot ensure the positivity of the estimation, it is sensitive to noise in the data and reflectance covariance matrix can only be approximated sub optimally (Urban et al, 2008). Therefore improvements proposed by different authors will depend on which problem wants to be solved (Shimano, 2006; Shen et al., 2007; Urban et al., 2008). For instance Urban et al. were focusing on the noise and proposing a spatially adaptive Wiener filter to estimate reflectance from images captured by a multispectral camera. A possible method of improvement that was considered in this paper was to introduce an optimization parameter that will deal with the noise and help achieving smoother reflectance curves.

Method

In order to perform evaluation of the mentioned algorithms, camera response was simulated with virtual camera sensor sensitivities (shown on Figure 1). The idea was to simulate the capture of an image with an RGB camera sensor and recover reflectance at each pixel by using the sensor's RGB response and some of the reflectance estimation algorithms. In order to do that, artificial sensor sensitivities were generated over the same range, 380 to 780 nm, and the capture was simulated under illuminant D65. A spectral image “fruitsandfowers.spb” was used as a test image. This image had size of 120 x 160 pixels and contains reflectance information at each pixel which was sampled with hyperspectral camera by 5 nm in the range of 380 to 780 nm.

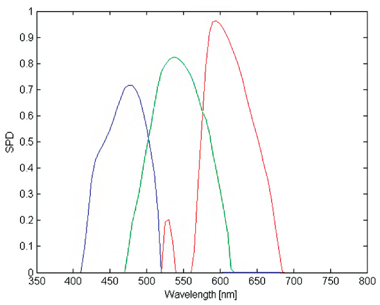

Figure 1. Spectral sensivity functions of a virtual RGB camera sensor

As a training set, Magbeth 24 patch Colour Checker was used. The results were evaluated visually and numerically. Visual evaluation was performed by the authors giving an opinion (*Excellent*, *Moderate*, *Poor*) about the recovered image compared to the original. In addition, random pixels were selected from the original image and their reflectances were compared with the estimations for the same pixels. The opinion was again stated as *Excellent*, *Moderate* or *Poor* recovery. For the numerical evaluation pixel wise difference of estimated reflectance to the original were calculated for each estimation method and the result was evaluated with the Frobenius norm. Colour differences computed as pixel wise Euclidean distance, $\Delta E_{00,}$ was also calculated between the original image and each estimated image under illuminat D65. For each of them the mean and the maximal differences were compared.

For the experiments the following data and notations were used:

- Spectral reflectance image I= “fruitsandfowers.spb”, with spatial dimensions: 120 x160 pixels and spectral dimension: 81.
- Sensitivity matrix W = [s_1 s_2 s_3] with dimensions 81 x 3, where columns correspond to discrete representation of RGB camera spectral sensitivity functions
- Matrix M = [m_1 …. m_{24}]^T with dimensions 24 x 81, where values in rows correspond to the Gretag Macbeth ColourChecker samples reflectances
- Diagonal matrix L with dimensions 81 x 81 represents the light source SPD. D65 was used to simulate capture.

All these elements are a function of a wavelength and they were sampled with a 5 nm sampling interval. Calculations were performed via Matlab software.

Recovery by using first three eigenvectors of the training set

In this paper the performance of the eigenvector method was tested by calculating the eigenvector basis on the autocorrelation matrix of Macbeth Colour Checker spectral data. Eigenvector decomposition of autocorrelation matrix was calculated using the Matlab function *eig*. As proposed by different authors (Connah et al., 2001; Dannemiller, 1992; Eem et al. 1994) the reflectance approximation was made by using only the first three eigenvectors that represent the highest variance of the training data. If the Colour Checker represents a set of most representative natural colours its eigenvectors should be able to reconstruct any reflectance coming from any other natural object. Based on the formulations provided by Kandi (Kandi, 2011) the following formulation was used for the calculation purposes in this paper:

$$\hat{r}_1 = UU^T r \qquad (2)$$

where is the estimated reflectance in one pixel position and is the original reflectance for the same pixel position and *U* corresponds to the matrix containing the first three eigenvectors from the eigenvector basis. The results for the whole image, pixel wise, were contained in the matrix.

In order to introduce the other algorithms it is necessary to introduce a camera model first (Solli et al, 2005). For the purpose of this paper camera was described as virtual, linear system with sensitivities defined it a matrix W containing three rows, one for each filter function. The light source used was defined as a diagonal matrix L. The camera response to the incoming signal can then be defined as:

$$x = W_L^{T} * r \qquad (3)$$

where x is the response of the camera and $W_L = W*L$ are the illumination weighted sensor sensitivities.

Orthogonal projection

Orthogonal projection algorithm involves usage of the orthogonal projection operator on the camera response data. It projects this response to the illumination weighted sensor sensitivity space called the sensor visual subspace (SVS) (Sharma, 1997). At this point the idea was to use this algorithm to evaluate the projection of a physical signal onto the SVS. For this purpose reflectance estimation was made by using the orthogonal projection pixel wise on the test image. The following formulation was used for that purpose:

$$\hat{r}_2 = W_L({W_L}^T W_L)^{-1}x \quad (4)$$

where is the estimated reflectance in one pixel position and is the camera response for the same pixel position. The results for the whole image pixel wise are contained in the matrix.

Wiener estimation

Wiener estimation requires an autocorrelation matrix generated from the training reflectance spectra. In this paper these reflectances were the Macbeth Colour Checker data. Reflectance estimation can be then made by using the Wiener estimation pixel wise over the whole image. Based on works of Urban et al. and Stigell et al. (Urban, 2008; Stigell et al, 2007) the following formulation for reflectance estimation based on Wiener approach was used in this paper:

$$\hat{r}_3 = \Sigma_3 W_L({W_L}^T \Sigma_3 W_L)^{-1}x \quad (5)$$

where is the estimated reflectance in one pixel position, is the camera response for the same pixel position and is the autocorrelation matrix. The results for the whole image pixel wise are contained in the matrix.

Optimized Wiener estimation

As mentioned before Wiener estimation can be optimized and for this purpose and optimization parameter ‘rho’ was introduced into the calculations to observe the quality of the estimation when this optimization is present. This parameter affects the smoothness of the obtained reflectance curve. The ‘rho’ value was changed in the range of 0-1 with the step of 0.1 and the changes in the smoothness of the recovered reflectance shape were observed. The value was set to 0.9 since it provided the best visual and numerical recovery results. A covariance matrix of first order autoregressive process that will be denoted as was generated with the defined parameter. The following formulation was used for that purpose:

$$\hat{r}_4 = \Sigma_4 W_L({W_L}^T \Sigma_4 W_L)^{-1}x \quad (6)$$

where is the estimated reflectance in one pixel position and is the camera response for the same pixel position. The results for the whole image pixel wise are contained in the matrix.

After obtaining all the necessary data the Frobenius norm was calculated using the Matlab function *norm(.,’fro’)* for all the pixel wise differences between the original and estimated reflectances. Colour differences were computed as pixel wise Euclidean distances ΔE_{00} between the original image and each estimated image under D65.

Results and discussion

The results of simulated images with all four methods are shown on Figure 2. PCA performs poor while all the other algorithms provided very good estimation. In the case of PCA approximation colours were too saturated and fine details were lost, which can be explained with the fact that only three eigenvectors were used for the recovery. The Wiener estimation performs very well, while the optimized version gives the most satisfactory results.

By selecting the random pixels in an image and plotting the corresponding reflectances the performance of each algorithm in estimating the spectral curves can be assessed. Figure 3 shows the results for one, randomly selected pixel.

From this example it can be seen that for the spectral shape recovery the PCA and the Wiener estimation provided the best results. When PCA was used some negative reflectance were noticed for other pixels in an image. These negative values can be explained by the nature of the eigenvectors and should not be neglected. It was also noticed that all the methods performs differently for particular pixels, due to the fact that the sensor used had low response at both ends of the visible range. This leads to losing some valuable information for the reconstruction. The orthogonal projection was shown to be the most sensitive to this, where there is no projection at the end of the spectrum at all.

Due to the variation in reflectance for different pixels the Frobenius norm was calculated. This norm provides the Mean Squared Error between the original image reluctance matrix R and the estimated reflectance matrixes. It provides information about how big is the overall difference between the original and estimated reflectances for each one of the methods. The results of comparing all the estimation with the original are presented in Table 1.

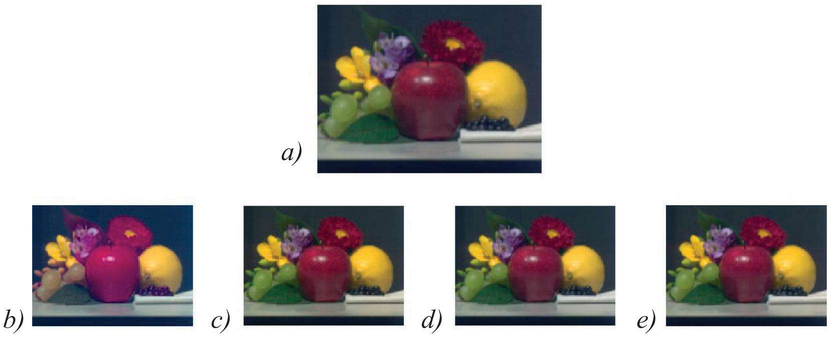

Figure 2. (a) Original image and the images obtained by (b) PCA approximation, (c) first algorithm used for reflectance estimation, (d) Wiener method, (e) optimized Wiener method

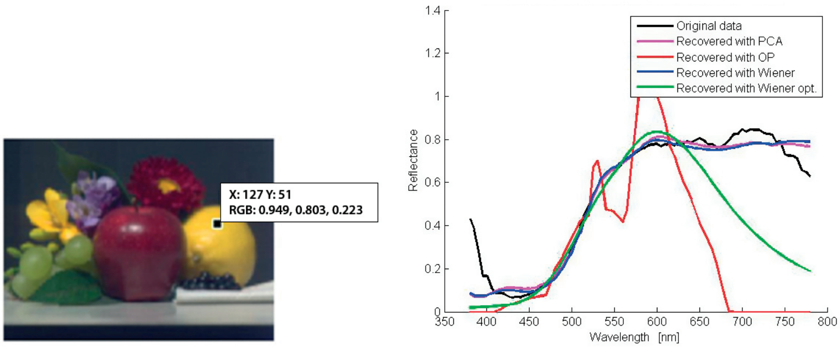

Figure 3. (a) Pixel position and (b) the comparison of the estimations of the spectral shape for each of the methods used

The F-norm values confirmed that from the spectral recovery point of view PCA gives the smallest difference compared to the original reflectance. This means that the spectral shape of the reflectance curves can be, for this particular case, estimated with the smallest error with PCA.

While the F-norm evaluates the results spectrally, another way to evaluate the results is colourimetrically, using the colour difference calculations. ΔE_{00} calculations were performed pixel wise over the whole image between the original and the estimated images. Maximal and mean value of the pixel wise difference was calculated. The results of colour difference evaluation are presented in Table 2.

From the results in Table 2 it can be seen that the Wiener estimate provided the image with the smallest overall colour difference. The highest difference is noticed if PCA was used, which is previously confirmed visually.

Conclusions

In this paper three most widely used alghoritms for recovering reflectance and PCA for estimating the reflectance from digital camera were evaluated. The aim was to define which of these methods can be used in graphic industry in order to replace spectrophotometric measurements with digital camera systems. High de-

Table 1. Frobenius norm for all the estimations compared to original

	PCA	Orthogonal projection	Wiener method	Optimized Wiener method
F-norm	102.25	254.51	137.51	190.85

Table 2. Colour difference (ΔE_{00}) for each estimation compared to original

	PCA	Orthogonal projection	Wiener method	Optimized Wiener method
Mean ΔE_{00}	11.82	4.05	1.95	2.33
Max ΔE_{00}	51.44	6.92	5.19	7.07

gree of accuracy in practical application was defined as a goal. It was noticed that even thou PCA provides the smallest difference in the spectral shape of the estimated reflectances it creates the biggest colour difference. Wiener method, on the other hand, provides the smallest colour difference but it does not perform the best in estimating the spectral shape if all the pixels in an image were taken into account. This suggests that in the case where reflectances had to be recovered from each pixel in an image it is possible to obtain different results in mean error values for reflectance recovery and colour accuracy. The optimized Wiener approach in this example didn't perform better than the classical Wiener approach. This is probably due to the fact that the virtual sensor had low response at both ends of the visible range, leading to some valuable information missing for the reconstruction. It could be that the parameter 'rho' used for the estimation performed badly in some parts of the image making the overall performance of this method not as good as the classical Wiener approach. The differences in F-norm value for PCA and Wiener method were minimal, hence it was concluded that Wiener method provide the best reflectance estimation while produces the best colour fit as well. If using Wiener estimate it is important to note that in order to achieve more accurate results sensor sensitivities must cover the whole visible range.

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