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Modelling KPIs of multi-product manufacturing systems prone to failures, scrap, and multiple reworks

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ABSTRACT

The evolving demands of modern industries have driven companies to implement flexible manufacturing systems capable of handling multiple product types. Nevertheless, the majority of studies conducted thus far to assess the performance of manufacturing systems are limited to single-product due to the complexity of handling multi-product environments. Moreover, the majority of these researches rarely consider the rework of non-compliant parts as a base for sustainable manufacturing. This paper presents a novel methodology to address the problem of assessing the performance of multiple-product manufacturing systems considering random breakdowns, scrap, and multiple rework attempts. A discrete-time Markov chain model is proposed to analyze the stochastic behavior of the manufacturing system. Analytical formulations are developed allowing to evaluate several key performance indicators such as throughput, effectiveness, availability, quality ratio, yield, scrapped parts, and required quantities of raw parts. A comprehensive simulation model mimics the stochastic and dynamic character of these flexible manufacturing systems is also designed. Extensive simulation experiments are carried out to validate the proposed analytical models across various production scenarios and system configurations, demonstrating the accuracy and robustness of the proposed approach and the analytical models.

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1. Introduction

The manufacturing industry faces growing pressure to advance performance and sustainability [1]. Driven by cost, quality, and environmental concerns, manufacturers are enhancing processes through Industry 4.0 and 5.0 innovations, such as AI, IoT, digital retrofitting, and flexible systems [1], [2]. These advancements improve automation and productivity while promoting waste reduction and resource efficiency [2]. A major challenge in modern manufacturing lies in managing process failures and non-conforming parts [3]. Specifically, defects, often caused by process variability or material inconsistencies, lead to inefficiencies that impact cycle times and overall system performance [3].

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Traditionally, manufacturers either scrap defective parts immediately [4], [5] or attempt to rework them to meet quality standards [6]. From the perspective of environmental responsibility, rework plays a crucial role in waste reduction and resource conservation, making it essential for sustainable manufacturing [6]. Despite the growing adoption of rework strategies by several industries, most of research primarily focus on scrapping non-compliant products, to overlooking the complexities of rework modelling [7], [8]. While some studies incorporate rework into performance assessments, they typically consider either a single rework attempt [9] or an infinite number of rework cycles [10]. All these studies are limited to a single-product manufacturing systems and cannot handle specific number of rework attempts for non-compliant parts.

This paper addresses this serious gap by modelling the impact of random failures, repairs, multiple-rework attempts, and scrap of non-compliant parts on the performance of flexible manufacturing systems producing several product types. Based on a discrete-time Markov chain methodology, numerous analytical formulae were developed to assess the system's Key Performance Indicators (KPIs) including effectiveness, throughput, availability, yield, scrapped parts, quality ratio, and required raw part quantities.

The remnant of this article is structured as follows. The next section analysis the literature on manufacturing system's performance under random disturbances caused by breakdowns, rework and scrap. Section 3 describes the studied multi-product manufacturing system. Sections 4 proposes a discrete-time Markov chain-based methodology to model several KPIs for multi-product flexible manufacturing systems prone to random failures, repairs, scrap, and specific multiple-rework attempts. A comprehensive simulation model is designed in Section 5, and used to complement the analytical framework. Numerical findings are discussed in Section 6. The work is finally concluded, and future research investigations are outlined in Section 7.

2. Literature review

In recent decades, extensive research on production systems has focused on evaluating and analyzing their performance under various disruptions and operational uncertainties [11], [12]. Among these, product quality issues are particularly critical, especially the handling of defective parts detected during inspections. These parts can either be immediately rejected [13], [14] or reworked to meet quality specifications [6], [9]. Rework, a critical element of quality, productivity, and sustainability management, involves corrective measures to repair or adjust non-compliant parts, ensuring they meet the specified requirements [15]. It is widely applied in industries such as semiconductors, steel, pharmaceuticals, food, and manufacturing [16]. By addressing defects, rework reduces material waste, improves resource efficiency, and enhances overall system performance. KPIs, such as throughput and Overall Equipment Effectiveness (OEE), are strongly influenced by the integration of defective products [3].

Several researchers have considered that a non-compliant product may undergo a single rework action, then it will be transformed to a conforming one. Davis and Kennedy [17], Pillai and Chandrasekharan [18], and Chao et al. [19] analyzed production lines subject to rework and reject employing absorbing Markov chains to assess fabrication costs. Biller et al. [20] and Jia et al. [9] examined rework loops in transfer lines, with intermediate stock constraints and machine breakdowns. In these models, each part undergoes inspection; non-conforming parts are reworked at the same production stage, not considering the likelihood of non-compliant parts rejection. Becker et al. [21] proposed a revised OEE formulation where each product is inspected and either accepted, rejected, or reworked once to meet specifications. In additive manufacturing context, Hajji et al. introduced an approximate OEE formulation for 3D printing, where non-compliant items are either reworked before acceptance or scrapped [22]. Arora et al. [6] developed an EPQ model to reduce backorder problems for battery industries considering single rework of imperfect batteries and outsourcing.

While all of these previous studies assume a single rework attempt, other researchers have considered that a non-complied item could go through an endless series of rework actions until they conform. Kimbler et al. [23] applied absorbing Markov chains to analyse a control process where parts exceeding tolerance limits undergo rework. Bowling et al. [24], Peng and Khasawne [25], and Nesaee and Nezhad [10] considered absorbing Markov chains to improve the process target for reliable fabrication systems subject to scrap and rework. Hadjinicola [26] studied a production line with reliable machines and where rework occurs at the previous station. Colledani and Angius [27] developed a platform for multi-stage fabrication lines with halfway stocks, integrating online inspection and quality monitoring. De Ron and Rooda [28] evaluated OEE for a reliable single machine where a non-compliant part is reworked again until it fulfils to specifications. Chen [29] analyzed OEE in a Taiwanese electronics company producing circuit boards under conditions involving rework, scrap, and failures. Wudhikarn [30] also evaluated OEE for a single machine subjected to failures and rework. Basak et al. [31] studied OEE in additive manufacturing through simulations, examining how product diversity and time constraints affect performance. Hajji et al. [32] assessed throughput in manufacturing cells where non-compliant items are either reworked or scrapped. Based on a simulation approach, Hajji et al. [33] compared two quality strategies: the first one letting to rework non-conforming items until they meet the specifications versus a second policy scrapping a non-conforming item despite it can be reworked.

In conclusion, all the previous studies consider only manufacturing systems producing a single product type and typically limited to two scenarios: (i) a single rework attempt, after which the defective part is considered acceptable, or (ii) an infinite number of rework cycles. The majority of the proposed formulae allow mainly the assessment of OEE of these production systems and rely on approximate methods, often based on general observations of manufacturing processes. However, while single-product manufacturing systems deliver high throughput, their limited adaptability blocks responsiveness to changing market demands. To overcome this, manufacturers are increasingly adopting flexible systems capable of handling multiple product types to adapt product variations and market dynamics. Thus, evaluating the performance of these flexible systems is crucial for understanding disruptions, enhancing overall performance, and developing efficient production plans and budgets. By examining important parameters like equipment performance, defect rates, and necessary raw materials, it assists companies in allocating resources effectively, cutting expenses, and increasing productivity in a changing market. To the best of our knowledge, no study has thoroughly analyzed the performance of flexible industrial systems producing several product types and prone to random failures, repairs, multiple reworks, and reject taking into account a limited and specific number of rework trials.

This paper's primary contributions are: (I) proposing a modelling method founded on discrete-time Markov chain describing the erratic and dynamic character of the flexible industrial system under a multiple rework quality policy, (II) developing analytical models to assess several KPIs including throughput, effectiveness, quality rate, yield, scrapped and required raw parts, and (III) designing a simulation model mimicking the real behavior of the flexible manufacturing system to validate the analytical results.

3. Manufacturing system description and notations

3.1 Manufacturing system overview and stochastic/dynamic behavior

This study examines a flexible manufacturing cell in which an automated machine handles production, inspection, and rework for various product types (Np different products) (Figure 1). A CNC machine, a PCB solder paste printer, or an additive manufacturing 3D printer can be considered as examples of this automated machine.

Each particular product type P_j ($j = 1, 2, ..., Np$) is produced in batches of L_j pieces as required by customers, with each part produced within a cycle time denoted as tc_j. Batches undergo cyclical processing (Figure 2) [34].

Each item in batch L_j undergoes conformity inspection within the automated machine. In fact, 100% inspection is increasingly favored over sampling for its reliability in defect detection and quality assurance [10], [25]. The inspection delay is part of the overall production time. After inspection, a produced item, belonging to batch L_j, might be in any of the listed states:

- Accepted: provided the item judged to be good and satisfies the necessary requirements.
- Reworked with probability α_j: provided the item is determined to be non-compliant but may be able to be made to comply with the prerequisite specifications.
- Rejected with probability β_j: provided the item is judged non- compliant and cannot be reworked.

For each product type, a non-conforming item can undergo a specified number of rework attempts (NR_j; $j = 1, 2, ..., Np$) depending on the industrial context and as established by departments of quality and manufacturing ($NR_j = 0, 1, 2, \ldots,$ or ∞). The rework process is performed on the production equipment requiring tr_j time units, which is equal to the cycle time tc_j.

The production cell is subject to accidental failures. The probability that the automated machine fails (be repaired) through a production/rework cycle, when producing a specific product P_j, equals p_j (r_j).

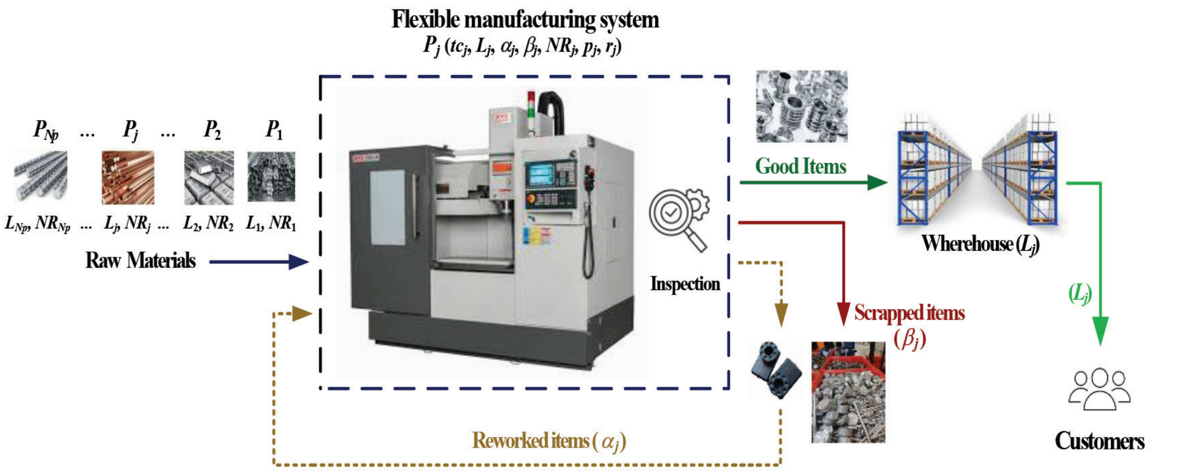

Figure 1. Flexible manufacturing system producing multi-products and prone to random failures, scrap, and rework

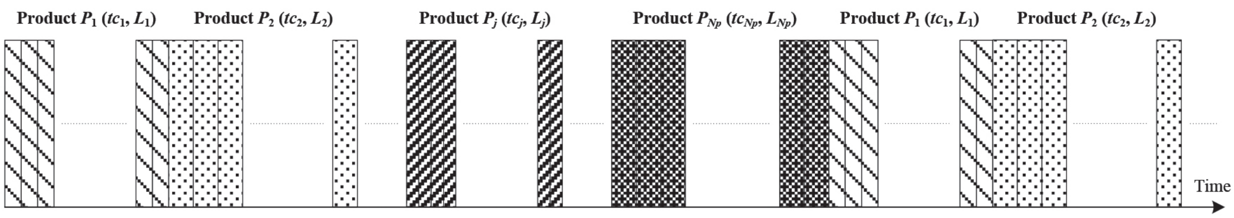

Figure 2. Workload of an Np products flexible manufacturing system

3.2 Notations

The notations used in this paper are listed below, where indices i, l, and j denote, respectively, the i^{th} attempt to rework a non-conforming item, a precise state where the production system is located, when producing the product type P_j. Further notations are defined in the text.

Np Number of part types produced by the flexible manufacturing cell.

tc_j Item processing time for product type P_j.

L_j Order batch quantity (required good parts) for product type P_j.

p_j machine failure probability during a production/rework cycle for product type P_j.

r_j Reparation probability during a production/rework cycle for product type P_j.

NR_j number of permitted rework try by item for product type P_j.

α_j probability of reworking non-conforming items for product type P_j.

β_j probability of scrapping non-conforming items for product type P_j.

Pr_l The steady state probability of residence of the flexible manufacturing cell in state l; where l = GP, F_{GP}, S, F_S, R_i, F_{Ri}, and $i = 1, 2, \ldots, NR_j$ (see Fig. 3).

E_j The effectiveness of the manufacturing cell when producing product type P_j.

QR_j The Quality Ratio when producing product type P_j.

Th_j The throughput when producing product type P_j.

$NbSP_j$ Quantity of rejected items produced through a specific period T_j for product P_j.

$NbRP_j$ The amount of raw items utilised through a specific period T_j for product type P_j.

Y_j The yield of manufacturing cell when producing product type P_j.

E The manufacturing cell overall effectiveness.

UTR The manufacturing cell operational availability.

QR The manufacturing cell overall quality ratio.

Th Overall production rate or throughput of the manufacturing cell (per time unit).

$NbSP$ Scrapped items produced during a specific period T.

$NbRP$ The raw items utilised by the manufacturing cell during a specific period T.

Y The flexible manufacturing cell overall yield.

4. Modelling methodology for the multi-product/multi-rework quality strategy

Consider the flexible manufacturing system described in section 3. The stochastic and dynamic behavior of the industrial system once producing a specific product type P_j can be described via the discrete-time Markov chain proposed in figure 3, where the cell can evolve from a specific state to the other according to a definite probability. When an item is discovered to be non-conforming to requirements and may be reworked, the manufacturing cell preconceives to rework it up to NR_j tries. Nonetheless, the produced item will be rejected and discarded if it is discovered to be non-conforming to requirements and cannot be reworked.

The Markov chain model consists of $2\times(NR_j+2)$ states, which are defined as follows:

- State GP: when the item is deemed good after inspection.
- State S: when the part is non-compliant and have to be discarded.
- State R_i ($i = 1, 2, \ldots, NR_j$): when the item is non-compliant and might undergo rework for the i^{th} attempt.
- Failure states: F_{GP}, F_S, and F_{Ri}, corresponding to states GP, S, and R_i, respectively.

To obtain the discrete-time Markov chain's steady-state probabilities, we resolve the steady-state equation system characterising the equilibrium state in which the state probabilities don't change throughout time [35]. These formulas come from the balance equations (1) to (7).

$$\begin{aligned} Pr_{\mathrm{GP}} = {} & (1-p_j)(1-\alpha_j-\beta_j)\left(Pr_{\mathrm{GP}} + \sum_{i=1}^{NR_j-1} Pr_{\mathrm{R}_i} + Pr_{\mathrm{S}}\right) \\ & + (1-p_j)Pr_{\mathrm{R}_{NR_j}} + r_j(1-\alpha_j-\beta_j) \\ & \left(Pr_{\mathrm{F_{GP}}} + \sum_{i=1}^{NR_j-1} Pr_{\mathrm{F}_{\mathrm{R}_i}} + Pr_{\mathrm{F_S}}\right) + r_j Pr_{\mathrm{F}_{\mathrm{R}_{NRj}}} \end{aligned} \tag{1}$$

$$Pr_{\mathrm{F_{GP}}} = p_j Pr_{\mathrm{GP}} + (1-r_j)Pr_{\mathrm{F_{GP}}} \tag{2}$$

$$\begin{aligned} Pr_{\mathrm{R}_1} = {} & (1-p_j)\alpha_j Pr_{\mathrm{GP}} + r_j\alpha_j Pr_{\mathrm{F_{GP}}} \\ & + (1-p_j)\alpha_j Pr_{\mathrm{S}} + r_j\alpha_j Pr_{\mathrm{F_S}} \end{aligned} \tag{3}$$

$$Pr_{\mathrm{R}_i} = (1-p_j)\alpha_j Pr_{\mathrm{R}_{i-1}} + r_j\alpha_j Pr_{\mathrm{F}_{\mathrm{R}_{i-1}}} \ ; \quad \text{for } i = 2, 3, \ldots, NR_j \tag{4}$$

$$Pr_{\mathrm{F}_{\mathrm{R}_i}} = p_j Pr_{\mathrm{R}_i} + (1-r_j)Pr_{\mathrm{F}_{\mathrm{R}_i}} \ ; \quad \text{for } i = 1, 2, \ldots, NR_j \tag{5}$$

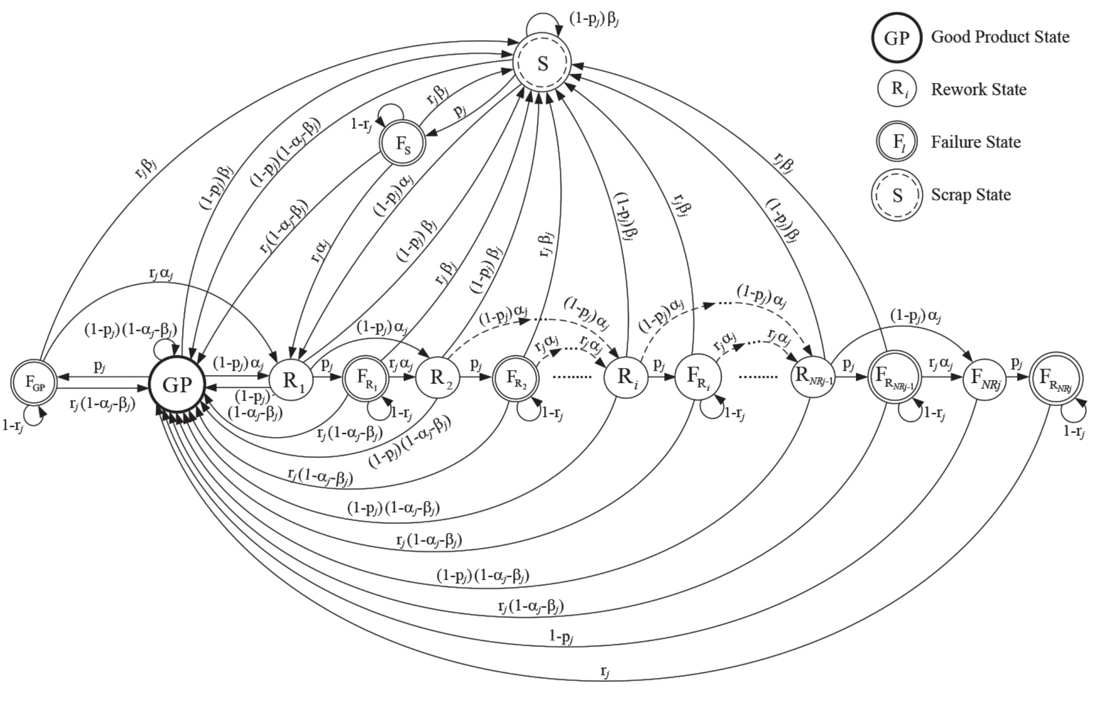

Figure 3. Flexible industrial system's Markov chain model while producing a specific product type P_j

$$Pr_{\mathrm{S}} = (1-p_j)\beta_j\left(Pr_{\mathrm{GP}} + \sum_{i=1}^{NR_j-1} Pr_{\mathrm{R}_i} + Pr_{\mathrm{S}}\right) + r_j\beta_j\left(Pr_{\mathrm{F_{GP}}} + \sum_{i=1}^{NR_j-1} Pr_{\mathrm{F_{R_i}}} + Pr_{\mathrm{F_S}}\right) \quad (6)$$

$$Pr_{\mathrm{F_S}} = p_j Pr_{\mathrm{S}} + (1-r_j)Pr_{\mathrm{F_S}} \quad (7)$$

This constitutes a set of linear equations who can be resolved in addition to the normalization requirement $\sum_l Pr_l = 1$, where l = GP, $\mathrm{F_{GP}}$, S, $\mathrm{F_S}$, R_i, $\mathrm{F_{R\mathit{i}}}$, and $i = 1, 2, \ldots, NR_j$. The $2\times(NR_j + 2)$ states' stationary probability are provided by equations (8) to (13).

$$Pr_{\mathrm{GP}} = \frac{r_j}{r_j+p_j}\frac{1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}{1-\alpha_j{}^{NR_j+1}} \quad (8)$$

$$Pr_{\mathrm{R}_i} = \frac{r_j}{r_j+p_j}\frac{\alpha_j(1-\alpha_j)}{1-\alpha_j{}^{NR_j+1}} \;\; ; \;\; \text{for } i = 1,2,\ldots,NR_j \quad (9)$$

$$Pr_{\mathrm{S}} = \frac{r_j}{r_j+p_j}\beta_j\left(1-\frac{\alpha_j{}^{NR_j}(1-\alpha_j)}{1-\alpha_j{}^{NR_j+1}}\right) \quad (10)$$

$$Pr_{\mathrm{F_{GP}}} = \frac{p_j}{r_j+p_j}\frac{1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}{1-\alpha_j{}^{NR_j+1}} \quad (11)$$

$$Pr_{\mathrm{F_{R_i}}} = \frac{p_j}{r_j+p_j}\frac{\alpha_j(1-\alpha_j)}{1-\alpha_j{}^{NR_j+1}} \;\; ; \;\; \text{for } i = 1,2,\ldots,NR_j \quad (12)$$

$$Pr_{\mathrm{F_{R_i}}} = \frac{p_j}{r_j+p_j}\frac{\alpha_j(1-\alpha_j)}{1-\alpha_j{}^{NR_j+1}} \;\; ; \;\; \text{for } i = 1,2,\ldots,NR_j \quad (13)$$

4.1 Key performance indicators when manufacturing a particular product type *Pj*

For most industrial systems, throughput is a crucial performance metric, representing the long-term average production rate. It is mainly influenced by the system effectiveness and the cycle time. For the studied manufacturing cell, when producing a specific product P_j, the throughput (Th_j) is given by equation (14) [34].

$$Th_j = E_j/tc_j \quad (14)$$

In this instance, the production cell effectiveness (E_j) is defined by the probability (Pr_{GP}) that the cell is in the GP state when producing product type P_j. It is expressed by equation (15).

$$E_j = \frac{r_j}{r_j+p_j}\frac{1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}{1-\alpha_j{}^{NR_j+1}} \quad (15)$$

The manufacturing cell availability (UTR_j) is calculated using equation (16), omitting epochs during which the cell is in a failure condition (F_l with l = GP, S, and R_i). One can note that the manufacturing cell availability depends only on the reliability and the maintainability parameters of the automated production machine and not on the products operating and quality parameters.

$$UTR_j = 1 - \sum_l Pr_{\mathrm{F}_l} = \frac{r_j}{r_j+p_j} \quad (16)$$

Given that system effectiveness equals the quality ratios multiplied by the availability, the quality ratio (Eq. 17) is deduced from equations (15) and (16).

$$QR_j = \frac{1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}{1-\alpha_j{}^{NR_j+1}} \quad (17)$$

The suggested Markov model may evaluate other *KPI*s including the number of rejected items (Eq. 18), the amount of raw parts needed (Eq. 19) through a specific period T_j, as well as the manufacturing yield (Eq. 20).

$$NbSP_j = \frac{T_j}{tc_j}\frac{r_j}{r_j+p_j}\beta_j\left(1-\frac{\alpha_j{}^{NR_j}(1-\alpha_j)}{1-\alpha_j{}^{NR_j+1}}\right) \quad (18)$$

$$NbRP_j = \frac{T_j}{tc_j}\frac{r_j}{r_j+p_j}\frac{1-\alpha_j}{1-\alpha_j{}^{NR_j+1}} \quad (19)$$

$$Y_j = 1 - \beta_j\frac{1-\alpha_j{}^{NR_j}}{1-\alpha_j} \quad (20)$$

It is important to note that since L_j represents the quantity ordered by the customer, the total number of required good parts for product type P_j corresponds to L_j. Hence, the required period (T_j) to produce a specific lot size of good parts (L_j) of product type P_j can be expressed by equation (21).

$$T_j = \frac{L_j}{Th_j} = \frac{L_j\, tc_j}{E_j} = \frac{L_j\, tc_j(r_j+p_j)\left(1-\alpha_j{}^{NR_j+1}\right)}{r_j\left(1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)\right)} \quad (21)$$

4.2 Key performance indicators for the Flexible Manufacturing System

To calculate the *KPI*s of this multi-product flexible manufacturing system, we consider that the system's behavior for each product type P_j is described using the Markov chain model proposed in Section 4.1. The manufacturing cell operates over a global time T to produce Np lot sizes of good parts for the required product types; T is the sum of the individual periods T_j required to produce a lot size (L_j) of good parts of a specific product type P_j. Since L_j represents the number of good parts to be produced for product P_j, the total quantity of good items made during the span T is the sum of all L_j, $j = 1, 2, \ldots, Np$.

The manufacturing cell overall throughput is the ratio of the produced Np lot sizes of good parts ($\sum_{j=1}^{Np} L_j$) by the total time T ($\sum_{j=1}^{Np} T_j$) where T_j is given by equation (21) :

$$Th = \frac{\sum_{j=1}^{Np} L_j}{\sum_{j=1}^{Np} T_j} = \frac{\sum_{j=1}^{Np} L_j}{\sum_{j=1}^{Np} L_j / Th_j} = \frac{\sum_{j=1}^{Np} L_j}{\sum_{j=1}^{Np} L_j\ tc_j \dfrac{1-\alpha_j{}^{NR_j+1}}{1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)} \dfrac{r_j+p_j}{r_j}}$$

Given that : $\sum_{j=1}^{Np} \frac{L_j\ tc_j\left(1-\alpha_j{}^{NR_j+1}\right)}{1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)} = \frac{L_1\ tc_1\left(1-\alpha_1{}^{NR_1+1}\right)}{1-\alpha_1-\beta_1\left(1-\alpha_1{}^{NR_1}\right)} + \cdots + \frac{L_{Np}\ tc_{Np}\left(1-\alpha_{Np}{}^{NR_{Np}+1}\right)}{1-\alpha_{Np}-\beta_{Np}\left(1-\alpha_{Np}{}^{NR_{Np}}\right)} = \frac{\sum_{j=1}^{Np} L_j\ tc_j\left(1-\alpha_j{}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1-\alpha_k-\beta_k\left(1-\alpha_k{}^{NR_k}\right)}{\prod_{j=1}^{Np} 1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}$

and $r_j + p_j / r_j = 1/UTR$, the overall throughput of the manufacturing cell will be expressed by equation (22).

$$Th = \frac{UTR \cdot \sum_{j=1}^{Np} L_j \cdot \prod_{j=1}^{Np} 1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j\ tc_j\left(1-\alpha_j{}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1-\alpha_k-\beta_k\left(1-\alpha_k{}^{NR_k}\right)} \tag{22}$$

Consequently, the effectiveness of the manufacturing cell is given by equations (23).

$$E = \frac{\sum_{j=1}^{Np} L_j\ \ tc_j}{\sum_{j=1}^{Np} T_j} = \frac{\sum_{j=1}^{Np} L_j\ \ tc_j}{\sum_{j=1}^{Np} L_j / Th_j} = \frac{UTR \cdot \sum_{j=1}^{Np} L_j\ \ tc_j \cdot \prod_{j=1}^{Np} 1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j\ tc_j\left(1-\alpha_j{}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1-\alpha_k-\beta_k\left(1-\alpha_k{}^{NR_k}\right)} \tag{23}$$

The overall quality ratio is given by equation (24) since the cell effectiveness is the multiplication of quality ratio (QR) and the up time ratio (UTR).

$$QR = \frac{\sum_{j=1}^{Np} L_j\ \ tc_j \cdot \prod_{j=1}^{Np} 1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j\ tc_j\left(1-\alpha_j{}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1-\alpha_k-\beta_k\left(1-\alpha_k{}^{NR_k}\right)} \tag{24}$$

The entire period T needed to produce the Np lot sizes of good parts for the entire product types is given by equation (25).

$$T = \sum_{j=1}^{Np} T_j = \frac{1}{UTR} \frac{\sum_{j=1}^{Np} L_j\ tc_j\left(1-\alpha_j{}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1-\alpha_k-\beta_k\left(1-\alpha_k{}^{NR_k}\right)}{\prod_{j=1}^{Np} 1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)} \tag{25}$$

The overall total quantity of scrapped items manufactured by the industrial cell as well as the required raw parts to manufacture the Np lot sizes of good parts for the entire product types are given by equations (26) and (27), respectively.

$$NbSP = \sum_{j=1}^{Np} NbSP_j = \sum_{j=1}^{Np} L_j\ \beta_j \frac{1-\alpha_j{}^{NR_j}}{1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)} = \frac{\sum_{j=1}^{Np} L_j\ \beta_j\left(1-\alpha_j{}^{NR_j}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1-\alpha_k-\beta_k\left(1-\alpha_k{}^{NR_k}\right)}{\prod_{j=1}^{Np} 1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)} \tag{26}$$

$$NbRP = \sum_{j=1}^{Np} L_j + \sum_{j=1}^{Np} NbSP_j = \frac{\sum_{j=1}^{Np} L_j\left(1-\alpha_j\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1-\alpha_k-\beta_k\left(1-\alpha_k{}^{NR_k}\right)}{\prod_{j=1}^{Np} 1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)} \tag{27}$$

Then, the overall yield is derived from the equation (28).

$$Y = \frac{\sum_{j=1}^{Np} L_j}{NbRP} = \frac{\sum_{j=1}^{Np} L_j \cdot \prod_{j=1}^{Np} 1-\alpha_j-\beta_j\left(1-\alpha_j{}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j\left(1-\alpha_j\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1-\alpha_k-\beta_k\left(1-\alpha_k{}^{NR_k}\right)} \tag{28}$$

4.3 Specific Cases

The general model developed in this paper can be adapted to a wide range of industrial contexts, leading to specific quality strategy considered in the literature such as the infinite rework attempts and the single rework cases.

The process of filling cans or bottles is a relevant example of unlimited rework attempts ($NR_j = \infty$ for $j = 1, 2, \ldots, Np$). During filling, defective items (underfilled, overfilled, or contaminated) are detected through quality control. These items are then emptied, potentially cleaned, and refilled. This operation can theoretically be repeated indefinitely until the product meets quality standards. This quality strategy has been widely studied in the literature [21, 24, 31, 32].

Several other studies have examined scenarios in which a faulty part is reworked once before becoming a conforming part [9, 19, 21, 22]. This situation is commonly observed in the automobile sector, for which body panels including paint flaws are frequently sanded and then repainted [36]. Such situations is modelled by assuming that the likelihoods of additional rework cycles are zero (α_j = 0, β_j = 0, and NR_j = 1).

5. Manufacturing System's simulation model

To validate and confirm the resilience of the suggested modelling methodology and the derived analytical formulae of the flexible manufacturing system's *KPI*s, a comprehensive simulation model that replicates the probabilistic and stochastic features of these production systems has also been designed with ARENA software [37].

Hundreds of experimentations were conducted by altering the number of manufactured product types (*Np*) and the manufacturing system parameters for each product type (tc_j, L_j, p_j, r_j, α_j, β_j, and NR_j). For each simulation and each system configuration, 10 independent replications are conducted, and each one runs for 10,000,000×*Np* time units, depending on the number of products being processed. A warm-up delay of 100,000 time units is included to ensure the consistency of the *KPI*s.

Figure 4 shows the flowchart of the discrete-event simulation model, thereby involving six main components:

1- INITIALIZATION module: specify product categories (*Np*), lot sizes (L_j), production durations (tc_j), breakdown (p_j) and repair (r_j) probabilities, scrapping (β_j) and reworking (α_j) probabilities, and the allowed number of reworks (NR_j) for each product type for each experiment. This phase also assigns the statistics that are cleared after the simulation warm-up time.

2- PRODUCTION module: monitors the flow of products and parts over the manufacturing cell. Before proceeding with the next product type (P_j+1), this module ensures that the cell produces exactly L_j good parts of the current product P_j. It also integrates the INSPECTION module that inspects every manufactured part to verify its conformity (see the following INSPECTION module). Seize and release actions of the machine are managed in this module.

3- INSPECTION module: inspects each completed part by sampling rework and scrap probabilities based on the product type (P_j) discrete probability distribution : α_j for rework state, β_j for scrap state, and (1- α_j - β_j) for good product state.

4- MAINTENANCE module: samples breakdown instants and repair periods for the industrial cell from their own probability distributions. Failures and repairs occur according to geometric distributions with respective parameters p_j and r_j.

5- TIME ADVANCE module: advances the time to the most imminent event according to the simulator clock and a discrete event scheduling plan.

6- UPDATE PERFORMANCE module: records the quantity of raw parts injected in the manufacturing cell (*NbRP*) and required to produce good parts (L_j) for each product type (P_j), the quantity of items being scrapped (*NbSP*), the quantity of rework actions executed during the simulation horizon, and the total time during which the manufacturing cell breaks down. This allows the assessment of the manufacturing cell effectiveness (*E*), availability (*UTR*), quality ratio (*QR*), yield (*Y*), and throughput (*Th*).

The simulation comes to an end if the simulation time TNOW crosses the simulation horizon.

6. Numerical results analysis and discussion

6.1 Analytical versus simulation results

Consider flexible manufacturing systems capable of handling *Np* different product types. To validate the proposed modelling methodology, hundreds of experiments were conducted by varying the number of product types (*Np*), and the operational, reliability, and quality parameters of each manufactured product *Pj* (tc_j, L_j, p_j, r_j, α_j, β_j, and NR_j).

Without loss of generality, table 1 presents a sample of the analysed cases of flexible manufacturing system that can produce up to 10 different products. While considering nevertheless realistic parameters, all the data have been randomly generated: $tc_j \in$ [10, 25 time units]; $L_j \in$ [600, 1000 parts]; $r_j \in$ [0.01, 0.35]; the failure probability is computed using the equation: $p_j = (r_j \times UTR)/(1 - UTR)$, where UTR = 70, 80, or 92%; $\alpha_j \in$ [0.01, 0.22]; $\beta_j \in$ [0.01, 0.15]; and $NR_j \in$ [1 to 8 rework attempts].

Three settings have been generated for each product type and denoted conf. 1, conf. 2, and conf. 3, according to a specific availability level of the au-

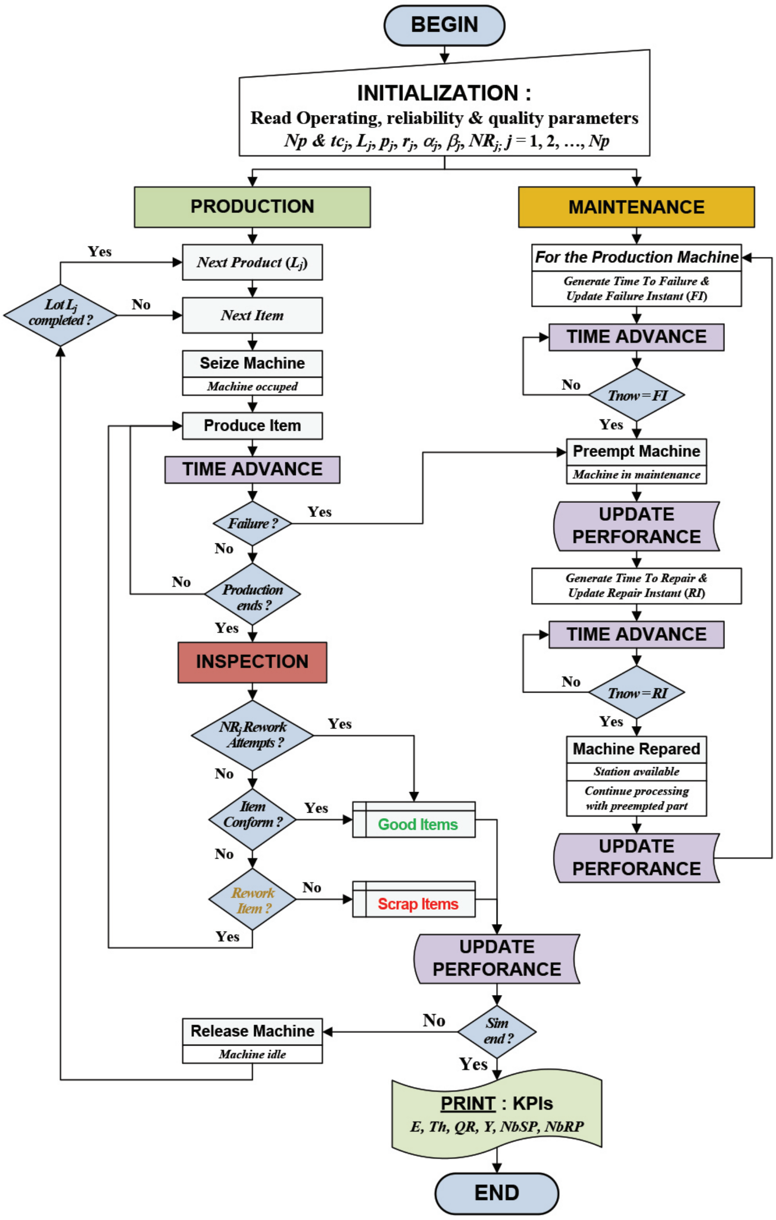

Figure 4. Manufacturing system general simulation model

Table 1. Operational, reliability, and quality parameters of the analyzed manufacturing cells

	P_1			P_2			P_3			P_4			P_5		
	conf.1	conf.2	conf.3	conf.1	conf.2	conf.3	conf.1	conf.2	conf.3	conf.1	conf.2	conf.3	conf.1	conf.2	conf.3
tc_j	21	12	17	11	19	13	15	18	23	10	18	12	22	20	11
L_j	953	637	816	745	921	718	879	914	983	649	884	728	795	681	647
r_j	0.1050	0.1600	0.0230	0.0550	0.2533	0.0176	0.0750	0.2400	0.0311	0.0500	0.2400	0.0162	0.1100	0.2667	0.0149
p_j	0.0450	0.0400	0.0020	0.0236	0.0633	0.0015	0.0321	0.0600	0.0027	0.0214	0.0600	0.0014	0.0471	0.0667	0.0013
α_j	0.08	0.21	0.07	0.03	0.16	0.08	0.12	0.05	0.13	0.05	0.1	0.05	0.09	0.06	0.03
β_j	0.07	0.12	0.11	0.05	0.04	0.05	0.05	0.07	0.07	0.03	0.08	0.1	0.04	0.05	0.05
NR_j	2	1	3	4	3	3	3	4	2	1	2	2	5	7	6

	P_6			P_7			P_8			P_9			P_{10}		
	conf.1	conf.2	conf.3	conf.1	conf.2	conf.3	conf.1	conf.2	conf.3	conf.1	conf.2	conf.3	conf.1	conf.2	conf.3
tc_j	14	23	11	12	10	15	22	20	11	16	19	10	13	18	21
L_j	986	820	885	758	675	832	900	963	991	734	891	965	623	931	786
r_j	0.0700	0.3067	0.0149	0.0600	0.1333	0.0203	0.1100	0.2667	0.0149	0.0800	0.2533	0.0135	0.0650	0.2400	0.0284
p_j	0.0300	0.0767	0.0013	0.0257	0.0333	0.0018	0.0471	0.0667	0.0013	0.0343	0.0633	0.0012	0.0279	0.0600	0.0025
α_j	0.07	0.09	0.12	0.13	0.07	0.09	0.14	0.09	0.15	0.17	0.12	0.09	0.03	0.02	0.11
β_j	0.09	0.06	0.08	0.09	0.14	0.04	0.08	0.11	0.03	0.1	0.5	0.11	0.05	0.07	0.13
NR_j	2	5	3	7	3	4	2	4	8	2	5	1	5	8	6

tomated manufacturing machine (Table 1); conf. 1, conf. 2, and conf. 3 correspond to a respective availability of 70% (low level), 80% (Intermediate level), and 92% (High level). For each manufacturing cell configuration, N simulation replications were conducted to collect N observations ($\overline{KPI}_{Sim}$). These results are then compared with those obtained from the proposed analytical models using the relative error equation (29).

$$\varepsilon\,(\%) = \frac{\overline{KPI}_{Sim} - KPI_{Anal}}{\overline{KPI}_{Sim}} \cdot 100 \tag{29}$$

where $\overline{KPI}_{Sim}$ and KPI_{Anal} represent, respectively, the simulated and the analytical values of a certain *KPI* (*Th*, *E*, *Y*, *QR*, *NbRP*, and *NbSP*).

According to these experiments (Tab. 1), table 2 highlights the *KPI*s derived from the proposed analytical formulae and the simulation model for the flexible manufacturing cell when producing 1, 2, 3, 5, 7, 9, and 10 different product types. It also provides the relative error of each *KPI* by contrasting the simulated and analytical findings (Eq. 29).

The findings in table 2 indicate that the proposed analytical models produce negligible relative errors for all *KPI*s when compared to the simulation results across all flexible manufacturing cell configurations. The mean relative error remains under 0.1% for all examined cases, regardless of the number of manufactured product types or the operational, quality, and reliability parameters.

To ascertain the precision and resilience of the proposed analytical equations, hundreds of randomly generated configurations where simulated and the analytical results where compared to simulation ones for each *KPI* through relative error evaluation (Eq. 29) and Student's t-test. The null hypothesis (H_0) assumes that a specific *KPI* derived from the analytical equation (KPI_{Anal}) is not significantly dissimilar from the *KPI* obtained from the simulation ($\overline{KPI}_{Sim}$). At a 1-δ confidence level, the null hypothesis is accepted if the t-distribution value with v degrees of freedom and a confidence level of 1-δ (i.e., $t_{v,1-\delta/2}$ for two-sided tests) exceeds the absolute value of t_0, as determined by equation (30).

$$t_0 = \frac{\overline{KPI}_{Sim} - KPI_{Anal}}{\sqrt{S^2/N}} \tag{30}$$

where $v = N - 1$, $\overline{KPI}_{Sim} = \frac{1}{N}\sum_{k=1}^{N} KPI_{Sim\,k}$, and $S^2 = \frac{1}{N-1}\sum_{k=1}^{N}(KPI_{Sim\,k} - \overline{KPI}_{Sim})^2$.

For $N = 10$ replications, and $\delta = 5\%$, the student's test table gives $t_{9,0.975} = 2.262$. Therefore, the hypothesis H_0 is accepted if $|t_0| \leq 2.262$.

The results show that the relative errors are negligible (less than 0.1%) and that the null hypothesis (H_0) is accepted for all configurations and all *KPI*s. This validates the proposed modelling methodology to assess the performance of multi-product flexible manufacturing systems, and confirms the accuracy and the robustness of all proposed *KPI*s analytical formulations (*E*, *Th*, *QR*, *Y*, *NbSP*, and *NbRP*).

6.2 *QR* versus Yield and its impact on the manufacturing system throughput

Unlike many researchers studying the productivity of manufacturing systems such as Nakajima [38],

Table 2. Analytical versus Simulation results

		E			*Th*			*QR*			*Y*			*NbSP*			*NbRP*		
Np		conf. 1	conf. 2	conf. 3	conf. 1	conf. 2	conf. 3	conf. 1	conf. 2	conf. 3	conf. 1	conf. 2	conf. 3	conf. 1	conf. 2	conf. 3	conf. 1	conf. 2	conf. 3
1	Analy.	0,5956	0,5818	0,7545	0,0284	0,0485	0,0444	0,8509	0,7273	0,8201	0,9244	0,8800	0,8818	78	87	109	1031	724	925
	Simul.	0,5955	0,5818	0,7538	0,0284	0,0485	0,0443	0,8507	0,7272	0,8194	0,9244	0,8801	0,8818	78	87	109	1031	724	925
	ε(%)	-0,0164	-0,0071	-0,0846	-0,0169	-0,0079	-0,0847	-0,0164	-0,0071	-0,0846	-0,0020	0,0106	0,0011	0,0259	-0,0884	-0,0096	0,0020	-0,0106	-0,0011
2	Analy.	0,6089	0,6215	0,7723	0,0367	0,0385	0,0511	0,8699	0,7768	0,8395	0,9348	0,9215	0,9106	118	133	151	1816	1691	1685
	Simul.	0,6087	0,6213	0,7721	0,0367	0,0385	0,0510	0,8696	0,7766	0,8393	0,9347	0,9214	0,9106	119	133	151	1817	1691	1685
	ε(%)	-0,0321	-0,0288	-0,0230	-0,0045	-0,0291	-0,0239	-0,0321	-0,0288	-0,0230	-0,0059	-0,0061	0,0039	0,0907	0,0772	-0,0431	0,0059	0,0061	-0,0039
3	Analy.	0,5998	0,6517	0,7554	0,0373	0,0387	0,0415	0,8568	0,8146	0,8210	0,9377	0,9233	0,9146	171	205	235	2748	2677	2752
	Simul.	0,5999	0,6516	0,7552	0,0374	0,0387	0,0415	0,8570	0,8146	0,8209	0,9376	0,9233	0,9146	171	205	235	2748	2677	2752
	ε(%)	0,0231	-0,0067	-0,0193	0,0233	-0,0058	-0,0190	0,0231	-0,0067	-0,0193	-0,0035	-0,0024	-0,0012	0,0557	0,0316	0,0146	0,0035	0,0024	0,0012
5	Analy.	0,6066	0,6637	0,7687	0,0373	0,0377	0,0485	0,8666	0,8297	0,8355	0,9464	0,9246	0,9163	228	329	356	4249	4366	4248
	Simul.	0,6067	0,6637	0,7689	0,0373	0,0377	0,0485	0,8667	0,8297	0,8358	0,9463	0,9246	0,9162	228	329	356	4249	4366	4248
	ε(%)	0,0100	0,0014	0,0327	0,0111	0,0016	0,0326	0,0100	0,0014	0,0327	-0,0034	-0,0009	-0,0077	0,0626	0,0120	0,0914	0,0034	0,0009	0,0077
7	Analy.	0,5969	0,6645	0,7693	0,0390	0,0380	0,0514	0,8527	0,8306	0,8362	0,9320	0,9161	0,9208	420	506	482	6185	6038	6091
	Simul.	0,5969	0,6645	0,7693	0,0390	0,0380	0,0514	0,8527	0,8307	0,8362	0,9320	0,9161	0,9209	421	507	482	6186	6039	6091
	ε(%)	-0,0073	0,0043	0,0014	-0,0068	0,0048	0,0010	-0,0073	0,0043	0,0014	-0,0010	-0,0015	0,0031	0,0154	0,0182	-0,0393	0,0010	0,0015	-0,0031
9	Analy.	0,5795	0,5745	0,7660	0,0358	0,0319	0,0555	0,8279	0,7182	0,8326	0,9241	0,8031	0,9222	608	1811	638	8007	9197	8203
	Simul.	0,5796	0,5746	0,7661	0,0358	0,0319	0,0555	0,8279	0,7182	0,8327	0,9240	0,8030	0,9223	608	1811	638	8007	9197	8203
	ε(%)	0,0081	0,0017	0,0103	0,0068	0,0010	0,0096	0,0081	0,0017	0,0103	-0,0032	-0,0038	0,0016	0,0422	0,0191	-0,0208	0,0032	0,0038	-0,0016
10	Analy.	0,5832	0,5884	0,7561	0,0366	0,0327	0,0522	0,8332	0,7355	0,8219	0,9259	0,8154	0,9154	642	1883	772	8664	10200	9123
	Simul.	0,5833	0,5884	0,7563	0,0366	0,0327	0,0522	0,8332	0,7355	0,8220	0,9259	0,8154	0,9154	642	1883	772	8664	10200	9123
	ε(%)	0,0082	0,0003	0,0188	0,0082	0,0031	0,0191	0,0082	0,0003	0,0188	-0,0027	-0,0018	0,0004	0,0366	0,0099	-0,0057	0,0027	0,0018	-0,0004

De Ron and Rooda [28], Wudhikarn [30], Sousa et al. [39], Jaqin et al. [40], Facchinetti and Citterio [41], and Stefana et al. [42] presuming that the quality ratio (*QR*) can be estimated by the manufacturing yield (*Y*) (Quantity of compliant items divided by Quantity of raw items), the proposed models, already validated by simulation, establish that the *QR* (Eq. 24) is significantly different from *Y* (Eq. 28) that whenever managers of quality and production division implement a rework strategy (see Table 2).

Appendix A demonstrates that the yield (*Y*) is strictly superior to the quality ratio (*QR*). Accordingly, considering the manufacturing yield instead of the quality ratio overestimates the real throughput and effectiveness of the flexible manufacturing systems. For instance, in the case where the system processes 2 (10) different product types and considering an availability of 80% (conf. 2) (70% (conf. 1)) (Tab. 2), the production system throughput assessed using the yield overestimates by more than 14% (8%) the effective throughput calculated from equation (22) based on the quality ratio.

In order to analyze the difference between *Y* and *QR*, consider the case where the manufacturing system processes 2 different product types (conf. 2) (Tab. 2). Table 3 illustrates the relative errors ε (Eq. 31) produced by contrasting the quality ratio to the yield when varying quality parameters (1) NR_1 and α_1 only for product 1, (2) NR_2 and α_2 only for product 2, and (3) NR_1, NR_2, α_1 and α_2 simultaneously for both products 1 and 2.

$$\varepsilon(\%) = \frac{Y - QR}{QR} \cdot 100 \tag{31}$$

The results show that this difference increases with the probabilities of rework attempt (α_j) and the allowed number of rework attempts (NR_j) that a non-conforming item may experience.

7. Conclusion

This paper introduces a novel modelling approach and analytical models for accurate assessing KPIs of multi-product flexible manufacturing systems susceptible to to random breakdowns and quality disturbances. The proposed models deal with situations in which the manufacturing system produces conforming parts while managing rework and scrap after inspection. The number of rework tries is predefined by the departments of quality and production.

A discrete-time Markov chain approach is employed to develop analytical models that capture various *KPI*s, including the system throughput, effectiveness, yield, quality ratio, and the amounts of discarded, and required raw items. From the proposed model, several specific cases relevant to different industrial contexts are derived, such as the 'infinite number of rework attempts' and the 'single rework attempt' scenarios.

In addition to the analytical models, a general simulation model was created, and hundreds of experiments have been conducted by varying the number of treated product types, and the products and equipment parameters, such as fabricating times, lot dimensions, breakdown and repair probabilities, probabilities of rework and rejection, together with the permitted number of rework tries. The results show that all proposed *KPI*s analytical models generate negligible errors, less than 0.1%, and that Student's *t*-test confirm that the *KPI*s derived from the analytical formulae are not significantly different from those obtained by the simulation model. These findings demonstrate the truthfulness and resilience of the proposed models.

This study moreover reveals that the quality ratio and yield are not comparable, in contrast to many system effectiveness studies that presume or estimate that they are. The findings indicate that the true throughput and effectiveness of these flexible

Table 3. Multi-product manufacturing system: Quality ratio (*QR*) versus yield (*Y*)

		NR_1				
α_1	*KPI*	1	2	3	4	5
0.14	*QR*	0,7917	0,7827	0,7814	0,7812	0,7812
	Y	0,9215	0,9139	0,9128	0,9127	0,9126
	$\mathcal{E}$ (%)	**16,39**	**16,76**	**16,81**	**16,82**	**16,82**
0.21	*QR*	0,7768	0,7603	0,7568	0,7561	0,7559
	Y	0,9215	0,9100	0,9076	0,9070	0,9069
	$\mathcal{E}$ (%)	**18,62**	**19,69**	**19,92**	**19,97**	**19,98**
0.28	*QR*	0,7625	0,7368	0,7296	0,7276	0,7270
	Y	0,9215	0,9061	0,9017	0,9004	0,9001
	$\mathcal{E}$ (%)	**20,85**	**22,97**	**23,58**	**23,76**	**23,80**

(1) varying NR_1 and α_1

		NR_2				
α_2	*KPI*	1	2	3	4	5
0.10	*QR*	0,8227	0,8156	0,8149	0,8148	0,8148
	Y	0,9256	0,9234	0,9232	0,9232	0,9231
	$\mathcal{E}$ (%)	**12,51**	**13,22**	**13,29**	**13,30**	**13,30**
0.16	*QR*	0,7943	0,7792	0,7768	0,7765	0,7764
	Y	0,9256	0,9221	0,9215	0,9214	0,9214
	$\mathcal{E}$ (%)	**16,53**	**18,33**	**18,62**	**18,67**	**18,67**
0.22	*QR*	0,7678	0,7429	0,7376	0,7364	0,7362
	Y	0,9256	0,9207	0,9197	0,9194	0,9194
	$\mathcal{E}$ (%)	**20,56**	**23,94**	**24,69**	**24,85**	**24,89**

(2) varying NR_2 and α_2

		NR 1 , *NR* 2				
(α_1 , α_2)	*KPI*	(1 , 1)	(2 , 2)	(3 , 3)	(4 , 4)	(5 , 5)
(0.14,0.10)	*QR*	0,8394	0,8220	0,8199	0,8196	0,8196
	Y	0,9256	0,9157	0,9144	0,9143	0,9143
	$\mathcal{E}$ (%)	**10,27**	**11,40**	**11,53**	**11,55**	**11,55**
(0.21,0.16)	*QR*	0,7943	0,7626	0,7568	0,7557	0,7555
	Y	0,9256	0,9106	0,9076	0,9069	0,9068
	$\mathcal{E}$ (%)	**16,53**	**19,41**	**19,92**	**20,01**	**20,03**
(0.28,0.22)	*QR*	0,7538	0,7062	0,6949	0,6920	0,6912
	Y	0,9256	0,9054	0,8999	0,8984	0,8980
	$\mathcal{E}$ (%)	**22,80**	**28,20**	**29,51**	**29,83**	**29,92**

(3) varying NR_1, NR_2, α_1, and α_2

manufacturing systems are overestimated by up to 14% when the production yield is taken into account rather than the quality ratio.

In actuality, precise evaluation of these manufacturing systems' performance is essential to efficient budgeting and production planning. It assists companies in effectively allocating resources, cutting expenses, and enhancing productivity in a dynamic market through the analysis of key performance metrics.

In the constantly evolving manufacturing landscape, this comprehensive approach to modelling irregular production, failures, repairs, and reworking of non-conforming components constitutes a significant step forward in achieving more precise and effective results. However, opportunities for further improvement remain. For example, extending this work to more complex flexible manufacturing systems - loading and unloading units, automatic tool change mechanism, and multi-machine manufacturing systems - could provide deeper insights and better enhance their realism and applicability in industrial contexts. Furthermore, more study should concentrate on extending the suggested models to take into consideration carbon emissions, energy-resource efficiency, and their environmental effects within the global framework of sustainability and Industry 5.0.

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Appendix A

For any product j with a rework probability α_j, we note that: $\alpha_j > {\alpha_j}^{NR_j+1}$ since $\alpha_j \in]0,1[$ and $NR_j \geq 1$ $(j = 1,2,\cdots,Np)$, so $1 - {\alpha_j}^{NR_j+1} > 1 - \alpha_j$. As $1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k}) > 0$, then $\left(1 - {\alpha_j}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k}) > \left(1 - \alpha_j\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})$.

Multiplying the two terms of this inequality by L_j, then the inverse of both terms satisfy the following inequality:

$$\frac{1}{L_j\left(1 - \alpha_j\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})} > \frac{1}{L_j\left(1 - {\alpha_j}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})}$$

Consequently, we have:

$$\frac{1}{\sum_{j=1}^{Np} L_j\left(1 - \alpha_j\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})} > \frac{1}{\sum_{j=1}^{Np} L_j\left(1 - {\alpha_j}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})}$$

Multiplying the 2 inequality sides by $\left(\sum_{j=1}^{Np} L_j \cdot \prod_{j=1}^{Np} 1 - \alpha_j - \beta_j\left(1 - {\alpha_j}^{NR_j}\right)\right)$, we get:

$$\frac{\sum_{j=1}^{Np} L_j \cdot \prod_{j=1}^{Np} 1 - \alpha_j - \beta_j\left(1 - {\alpha_j}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j\left(1 - \alpha_j\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})} > \frac{\sum_{j=1}^{Np} L_j \cdot \prod_{j=1}^{Np} 1 - \alpha_j - \beta_j\left(1 - {\alpha_j}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j\left(1 - {\alpha_j}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})}$$

Multiply the inequality second term numerator and denominator by the cycle time tc, considering that $tc_j = tc$, we got:

$$\frac{\sum_{j=1}^{Np} L_j \cdot \prod_{j=1}^{Np} 1 - \alpha_j - \beta_j\left(1 - {\alpha_j}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j\left(1 - \alpha_j\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})} > \frac{tc\sum_{j=1}^{Np} L_j \cdot \prod_{j=1}^{Np} 1 - \alpha_j - \beta_j\left(1 - {\alpha_j}^{NR_j}\right)}{tc\sum_{j=1}^{Np} L_j\left(1 - {\alpha_j}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})}$$

Finally we got:

$$\frac{\sum_{j=1}^{Np} L_j \cdot \prod_{j=1}^{Np} 1 - \alpha_j - \beta_j\left(1 - {\alpha_j}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j\left(1 - \alpha_j\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})} > \frac{\sum_{j=1}^{Np} L_j \cdot tc_j \cdot \prod_{j=1}^{Np} 1 - \alpha_j - \beta_j\left(1 - {\alpha_j}^{NR_j}\right)}{\sum_{j=1}^{Np} L_j \cdot tc_j\left(1 - {\alpha_j}^{NR_j+1}\right)\prod_{\substack{k=1\\k\neq j}}^{Np} 1 - \alpha_k - \beta_k(1 - {\alpha_k}^{NR_k})}$$

The left side term is the expression of the yield (Eq. 28), and the right side term is the expression of the quality ratio (Eq. 24). So, we can conclude that $Y > QR$.