Journal for Technology of Plasticity, Vol. 33 (2008), Number 1-2

PRESSURE DETERMINATION OF THE BACKWARD CUP EXTRUSION PROCESS

Velibor Marinković
University of Niš, Faculty of Mechanical Engineering, Serbia

ABSTRACT

Extreme loading of the tool, especially of the punch, sometimes represents a limiting factor in applying the backward cup extrusion process. If the punch is not properly centered and guided, each disturbance can lead to instability of the extrusion process and damage or breaking of the punch. That is why it is of primary importance to calculate pressure (force) of the punch. To calculate the punch pressure, an approximate energetic method known as the upper bound method is applied in this study. For easier application in engineering practice the derived formula is presented in the form of diagram. Likewise, to meet the needs of design and calculation of the technological processes of forming by means of computer, an approximate formula is also proposed in the paper. It has been found that the discrepancy of the theoretical and experimental results is within the boundaries of acceptable error.

Key words: *Cold Forming, Backward Extrusion, Extrusion Pressure, Upper Bound Method*

1. INTRODUCTION

Cold forming processes (especially those of extrusion) are specially effective and important for the manufacturing of parts in large series. These processes are characterized by high productivity, economical approach, surface quality, dimension accuracy as well as improved mechanical properties of materials. Because of their numerous advantages, the cold forming processes find the widest application in the manufacturing of automobile parts (parts of motor, transmission and driving mechanism), aircraft, motorcycles; of elements for general application (nuts, bolts, tubes, cans, shafts, gears), as well as elements for special application (ball-bearing rings, cup ram for hydraulic valves, spot-welding electrodes, chuck wrenches, claws, sleeves, oil-cups, hollow profiles). For the making of some elements such as two-wall cups or hexagonal semi-hollow nuts and pins there is no alternative or more economical process than extrusion [1-4]. Due to all the

above-said, the cold forming process is considered as a representative of all bulk forming processes.
The basic operations of the cold extrusion are forward extrusion, backward extrusion and radial extrusion. There are also combined processes of extrusion in which the billet is extruded simultaneously in the forward and backward directions as well as radially through many orifices in the tool. The processes of combined extrusions aim at reducing the number of stages otherwise required for forming parts of more intricate shapes.
The backward cup extrusion is applied as one of the primary operations or as a step in multi-stage forming process. Of very efficient application is that of the backward cup extrusion in the processes of combined cold forming, especially in different variants of combined extrusion [2], [5-7]. The cold backward cup extrusion is mostly applied to the making of cups of constant or variable wall thickness. For the making of such elements we can use other forming processes such as deep drawing and/or ironing. Comparing to these processes, the backward cold extrusion has considerable techno-economic advantages despite considerable investments in tools and machines (presses), since the processes of deep drawing and ironing are realized, as a rule, in many operations.
The backward cup extrusion is undoubtedly one of the most critical operations of cold forming due to a high loading of the punch and extremely severe tribological conditions. Such tribological conditions can cause the breaking of the lubricant film, sticking of the workpiece material to the tool and thus the damaging of the cup surface and significant tool wear. Altan with his co-workers [8], [9] has come upon the idea to apply a double cup extrusion test (backward cup extrusion/forward cup extrusion) to evaluation of frictional conditions in cold forming operations. The tools for backward extrusion are exposed to high contact stresses which, in special cases, can be a limiting factor in the application of this process of extrusion.
For optimal design of both extrusion process and respective tools the experts in practice need to be provided with sufficiently reliable and accurate methods for calculating the load of the tool working parts (punch, die). The determination of the punch pressure in the process of backward cup extrusion represents the most important segment of the whole calculation. On the basis of the calculated punch pressure it is easy to determine extrusion force and deformation work which are both the basic parameters for the selection of the deformation machine (press) for each forming operation.

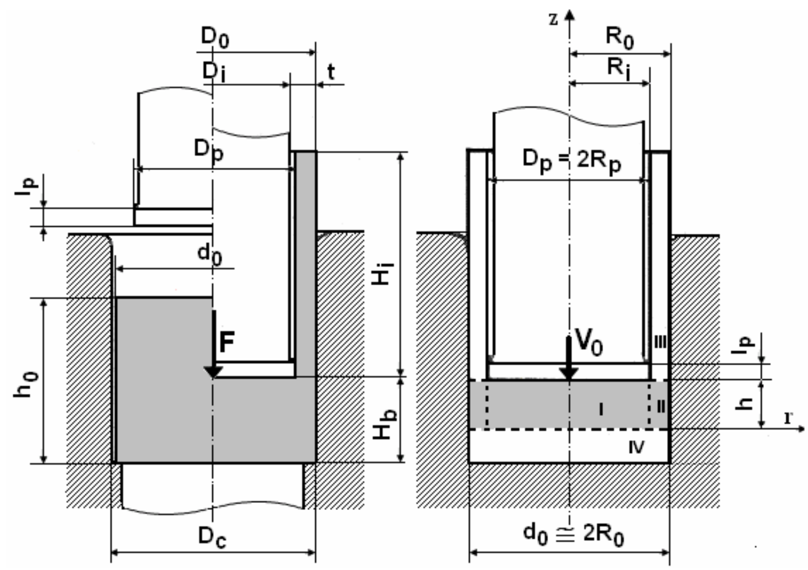
2. ANALYSIS OF BACKWARD CUP EXTRUSION PROCESS

One of the most important and most frequently applied processes of extrusion in industry is that of backward extrusion. A diagram of this extrusion process is given in Fig. 1a.
This procedure is characterized by a complex stress/strain state and non-steady material flow. Both these facts are confirmed by numerous experiments. That is why this extrusion process has been the subject of research of many authors [5], [10-17]. However, as has properly been observed by Storoschev [13], the complexity of the problem has led to the solutions that are very little adjusted to each other. Some of them are not sufficiently confirmed in the experimental way while others, due to the calculation complexity, are not acceptable to experts in industry. The aim of this research is to come to the conclusions which will be sufficiently accurate, on one hand, and, on the other, sufficiently simple for practical application.

2.1. Material Flow

Prior to extrusion, the die orifice is to have inserted a cylindrical billet of diameter d_0 and height h_0, while the billet diameter has to be smaller than the die diameter ($D_c - d_0 \cong 0.2$ (mm)). The intrusion of the punch nose into the material marks the beginning of the backward cup extrusion process.
The first phase is characterized by axial material upsetting under the punch which causes an increase of the billet diameter until the clearance between the billet and the die is eliminated after which the material flow is directed towards the ring orifice between the punch and the die ($c = D_c - D_p$) , thus including the punch land (l_p).

With further punch intrusion, the material continually flows in the sense opposed to the direction of the punch movement, along the punch land, along the die wall and freely above it outside. This is a pseudo-steady phase of the material flow which is, as a rule, the most time consuming. In this phase the zone of plastic deformations is formed (ZPD)



a) b)

Figure 1 – General Scheme of Backward Cup Extrusion:
a) Process Geometry; b) ZPD Discretization in Longitudinal Plane of Workpiece

This is a part of the billet volume which is subjected to plastic deforming while it spreads to a certain distance from the punch nose by height and to the die walls along the width of the longitudinal section of workpiece. ZPD is of irregular geometric shape. The longitudinal section of workpiece is most often approximated by parallelogram of length $D_c \cong d_0$ and height h (Fig. 1b). Both the theoretical and experimental results show that height of the ZPD depends on deformation degree and tribological conditions [5], [11-16]. The material outside the ZPD behaves like a rigid body and does not undergo any deformation.

In the last phase of extrusion, with the punch deeply intruded into the material, there is a possibility for the thickness of the cup bottom to become smaller than the ZPD height h (Fig. 1). Experiments point to the force's increase on the punch with respect to the previous extrusion phases. The force at the end of the extrusion process can be greater than the maximal which most often emerges at the beginning of the second extrusion phase. This phenomenon can be explained by intensive material compression under the punch and disturbed material flow from the die.
Finally, we should mention that, due to elastic deforming of the punch and the matrix, certain deviation of the cup diameter from the given values ($D_0 \cong D_c \ ; \ D_i \cong D_p$) takes place, but these differences are theoretically and practically negligible.

2.2. Kinematically Admissible Velocity Field

As has already been mentioned, in backward cup extrusion, the deformation process is localized on the ZPD. It can be accepted that the ZPD, of dimensions $2R_0 x\, h$, moves in the direction of the punch's movement, together with it, without changing either shape or dimensions (Fig. 1b).
Since it is an axi-symmetrical forming process, it is suitable to adopt polar-cylindrical coordinate system (r, θ, z). For the selected coordinate system, regarding the nature of the material flow, the imposed division of the ZDP is that into two regions, namely – cylindrical region I and ring region II. The billet material above the ZDP (region III) and under the ZPD (region IV) is subjected only to elastic deformations which are negligible. The punch and the die are treated as rigid bodies.
Most of the researchers start from the assumption about homogeneity of local (elementary) deformations in the ZDP, that is, from the assumption that component velocities V_r and V_z in the direction of coordinate axes r and z are functions of only intrinsic coordinates (most often linear). In doing so, it is assumed that from the conditions of axial symmetry it must be $V_\theta = 0$. More complex functions for component velocities have prevented the acquisition of the solution in an analytical form.
On the basis of the known theoretical solutions, numerous experimental research studies on models and real materials and the already adopted form of the ZDP, we can, for component functions in regions I, II, III i IV adopt the following functions (Fig. 1b):

$$V_r = \frac{V_0}{h} r \left(1 - \frac{y}{h}\right) \quad ; \quad V_z = -2\frac{V_0}{h} z \left(1 - \frac{y}{2h}\right) \tag{1a}$$

$$V_r = -\frac{V_0}{h} \frac{R_p^2}{R_0^2 - R_p^2} r \left(1 - \frac{z}{h}\right)\left(1 - \frac{R_0^2}{r^2}\right) \quad ; \quad V_z = 2\frac{V_0}{h} \frac{R_p^2}{R_0^2 - R_p^2} z \left(1 - \frac{z}{2h}\right) \tag{1b}$$

$$V_r = 0 \quad ; \quad V_z = V_1 = V_0 \frac{R_p^2}{R_0^2 - R_p^2} \tag{1c}$$

$$V_r = 0 \quad ; \quad V_z = 0 \tag{1d}$$

where: V_0 - punch (ram) velocity.

The continuity condition for isotropic and non-compressible material can be represented in the form:

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = \frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z} = 0 \qquad (2)$$

It is easy to prove that condition (2) is satisfied for all the regions of the ZPD.
Likewise, all the boundary and transition conditions within the ZPD are satisfied:

$$\begin{aligned} &V_{zI} = -V_0 \ , \ for \ z = h \ ; V_{rI} = 0 \ , \ for \ r = 0 \ ; V_{zI} = V_{zII} = 0 \ , \ for \ z = 0 \ , \\ &V_{zII} = V_1 \ , \ for \ z = h \ ; V_{rII} = 0 \ , for \ r = R_0 \ , V_{rII} = V_{rI} \ , \ for \ r = R_p \end{aligned} \qquad (3)$$

The components of the deformation velocity tensor for axi-symmetrical deformation are determined from formula [18-21]:

$$\dot{\varepsilon}_r = \frac{\partial V_r}{\partial r} \ ; \ \ \dot{\varepsilon}_\theta = \frac{V_r}{r} \ ; \ \ \dot{\varepsilon}_z = \frac{\partial V_z}{\partial z} \ ; \ \ \dot{\varepsilon}_{rz} = \frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r} \ ; \ \ \dot{\varepsilon}_{r\theta} = \dot{\varepsilon}_{z\theta} = 0 \qquad (4)$$

For further calculation it is necessary to determine equivalent shear strain rate [18-21]:

$$\dot{\gamma}_e = \sqrt{4\left(\dot{\varepsilon}_r^2 + \dot{\varepsilon}_\theta^2 + \dot{\varepsilon}_r \, \dot{\varepsilon}_\theta\right) + \dot{\varepsilon}_{rz}^2} \qquad (5)$$

On the basis of formulas (1), (4) and (5) for regions I and II it is obtained that:

$$\dot{\gamma}_e = \frac{V_0}{h^2}\sqrt{12\left(h-z\right)^2 + r^2} \qquad (6a)$$

$$\dot{\gamma}_e = \frac{V_0}{h^2}\frac{R_p^2}{R_0^2 - R_p^2}\sqrt{4\left(h-z\right)^2\left(3 + \frac{R_0^4}{r^4}\right) + r^2\left(\frac{R_0^2}{r^2} - 1\right)^2} \qquad (6b)$$

3. APPLICATION OF THE UPPER BOUND METHOD

On the basis of so-called extremal theorems of the ideal plasticity theory which were first formulated by Prager and Drucker [18-20], Upper Bound Method (UBM) has been developed and widely applied to the problems of metal forming by Johnson, Kudo and Kobayashi [18-20] followed by many others. Bramley and co-workers [21], [22], have successfully developed and used the Upper Bound Elemental Technique (UBET) for modelling numerous manufacturing processes such as closed-die forging and extrusion.

UBM is based on the use of a kinematically admissible field of velocities within the ZPD and reological model of rigid-plastic body. By its essence, it belongs to approximate methods of calculation and it gives enlarged values for force and deformation work. Deformation force is obtained from the equality of power of the outer forces and the total power consumption.

The total power dissipation ($\dot{W}_t$) for recommended field of velocities, can be expressed in the form [18-20]:

$$\dot{W}_t = \dot{W}_i + \dot{W}_s + \dot{W}_f = \sum k \iiint_V \dot{\gamma}_e dV + \sum \iint_{A_s} \tau_s \left|\Delta V_s\right| dA_s + \sum \iint_{A_f} \tau_f \left|\Delta V_f\right| dA_f \qquad (7)$$

where: $\dot{W}_i$ - internal (ideal) power dissipation due to pure plastic deformation, $\dot{W}_s$ - internal shear power dissipation due to velocity discontinuities along boundaries of regions of ZPD, $\dot{W}_f$ - frictional power dissipation due to friction at interfaces between workpiece and tool, k - internal shear stress ($\tau_s = k$), ΔV_s - velocity discontinuity at the shear boundaries, ΔV_f - velocity discontinuity at the tool/material interface, τ_f - friction shear stress.

Between contact shear stress τ_f and friction coefficient μ there is proportional dependence. In the field of bulk forming the most often used is the Prandtl-Siebel relation:

$$\tau_f = mk \qquad (8)$$

where: m - friction factor ($m = \sqrt{3}\mu$).
According to the Huber-Mises criterion of plasticity, the following relation exists:

$$k = \sigma_e / \sqrt{3} \qquad (11) \qquad (9)$$

where: σ_e - flow stress (effective stress).
The internal power dissipation is obtained as the sum of the forces in region I and region II:

$$\dot{W}_{iI} = k \int_0^{2\pi}\int_0^{h}\int_0^{R_1} \dot{\gamma}_{eI}\, rdrdzd\theta \quad ; \quad \dot{W}_{iII} = k \int_0^{2\pi}\int_0^{h}\int_{R_1}^{R_0} \dot{\gamma}_{eII}\, rdrdzd\theta \qquad (10)$$

The internal shear power dissipation is obtained as the sum of the shear forces at the boundaries of regions I/II, I/IV i II/IV:

$$\dot{W}_{sI/II} = k \int_0^{2\pi}\int_0^{h} \left|\Delta V_z\right| R_p dzd\theta \; ; \; \dot{W}_{sI/IV} = k \int_0^{2\pi}\int_0^{h} \left|\Delta V_r\right| rdzd\theta \; ;$$

$$\dot{W}_{sII/IV} = k \int_0^{2\pi}\int_{Rp}^{R_0} \left|\Delta V_r\right| rdzd\theta \qquad (11)$$

Friction losses emerge due to material motion along the die wall and through the punch land. Neglecting friction on the die outside the ZPD, according to equation (7), it is obtained that:

$$\dot{W}_{fc} = 2\mu_0 k \int_0^{2\pi}\int_0^{h} \left|\Delta V_z\right| R_0 dzd\theta ; \quad \dot{W}_{fp} = 2\mu_1 k \int_0^{2\pi}\int_0^{l_p} V_a\, R_1 dzd\theta \qquad (12)$$

For practical reasons, for further calculations it is assumed that friction coefficient is constant and that it has the same values on all the contact surfaces ($\mu_0 \cong \mu_1 \equiv \mu$).
Integrals (10), (11) and (12) are simplified by integralling by coordinate θ since, by the symmetry condition, the subintegral functions do not depend on this coordinate.
On the basis of the above-stated relations for total reduced deforming power we obtain:

$$\begin{aligned}
\bar{\dot{W}}_t &= \frac{\dot{W}_t}{kA_pV_0} = \frac{1}{\bar{h}\,R_p^2}\int_0^{R_p}\left[rU_1 + \frac{\sqrt{3}r^3}{6\bar{h}R_p^2}\ell n\frac{U+2\sqrt{3}\bar{h}R_p}{r}\right]dr + \\
&+\frac{\psi}{1-\psi}\frac{1}{R_p^2}\int_{R_p}^{R_0}\left(\frac{1}{r}\sqrt{4U_2+\frac{r^2}{\bar{h}R_p^2}U_3}+\frac{rU_3^2}{2\bar{h}^2R_p^2}\sqrt{U_2}\,\ell n\frac{2\bar{h}R_p\sqrt{U_2}+\sqrt{4\bar{h}^2R_p^2U_2+r^2U_3^2}}{rU_3}\right)dr + \\
&+\frac{4}{3}\frac{\bar{h}}{1-\psi}+\frac{2}{3\bar{h}}+\frac{2}{\bar{h}(1-\psi)}\left(\frac{2}{3\sqrt{\psi}}+\frac{\psi}{3}-1\right)+\frac{8}{3}\mu\frac{\bar{h}\sqrt{\psi}}{1-\psi}+4\mu\frac{\psi}{1-\psi}\frac{l_p}{R_p} \\
&\left\{U_1=\sqrt{r^2+12\bar{h}^2R_p^2}\ ;\ U_2=3r^4+R_0^4\ ;\ U_3=R_0^2-r_2\right\}
\end{aligned} \tag{13}$$

where: $\bar{h}$ - relative height ZPD ($\bar{h}=h/R_p$), A_p - punch nose surface, ψ - reduction ratio (ratio between the cross-sections of punch nose and container).
Integrals in relation (13) cannot be solved in the closed form but by applying some of the well-known procedure of numerical calculation. In this case the analysis has shown that some of the members of the subintegral functions can be neglected without having any essential impact on the integral value. Then relation (13), after integralling and certain transformations, is reduced to the form:

$$\begin{aligned}
\frac{\dot{W}_t}{kA_pV_0} &= \frac{1}{3\bar{h}}\left[\sqrt{(1+12\bar{h}^2)^3}-24\sqrt{3}\bar{h}^3\right]+\frac{1}{1-\psi}\left[2-\sqrt{1+3\psi^2}+\ell n\frac{1+\sqrt{1+3\psi^2}}{3\psi}\right]+\frac{2}{3\bar{h}}\left[1+\frac{2\bar{h}^2}{1-\psi}\right]+ \\
&+\frac{2}{(1-\psi)\bar{h}}\left[\frac{2}{3\sqrt{\psi}}+\frac{\psi}{3}-1\right]+\frac{8}{3}\mu\frac{\sqrt{\psi}}{1-\psi}\bar{h}+4\mu\frac{\psi}{1-\psi}\frac{l_p}{R_p}
\end{aligned} \tag{14}$$

In relation (14) there emerges currently unknown value $\bar{h}$, which can be determined from the conditions for minimum of total energy dissipation. This procedure is also characteristic for some other methods of the general theory of plasticity..
Accordingly, from condition $d\dot{W}_{tot}/d\bar{h}=0$ it is obtained that:

$$\begin{aligned}
&4\sqrt{(1+12\bar{h}^2)^3}\left\{\bar{h}^2\left(\frac{1}{1-\psi}+2\mu\frac{\sqrt{\psi}}{1-\psi}-12\sqrt{3}\bar{h}\right)-\left[\frac{1}{2}+\frac{1}{1-\psi}\left(\frac{1}{\sqrt{\psi}}+\frac{\psi}{2}-\frac{3}{2}\right)\right]\right\}+ \\
&+(1+12\bar{h}^2)^2(24\bar{h}^2-1)=0
\end{aligned} \tag{15}$$

Equation (15) is possible to solve only by numerical procedure for given parameters μ and ψ. Solution in the form of function $\overline{h} = f(\psi)$, for $\mu = 0.05$ and $\mu = 0.25$ is given graphically in Fig. 2 (denoted with broken lines).

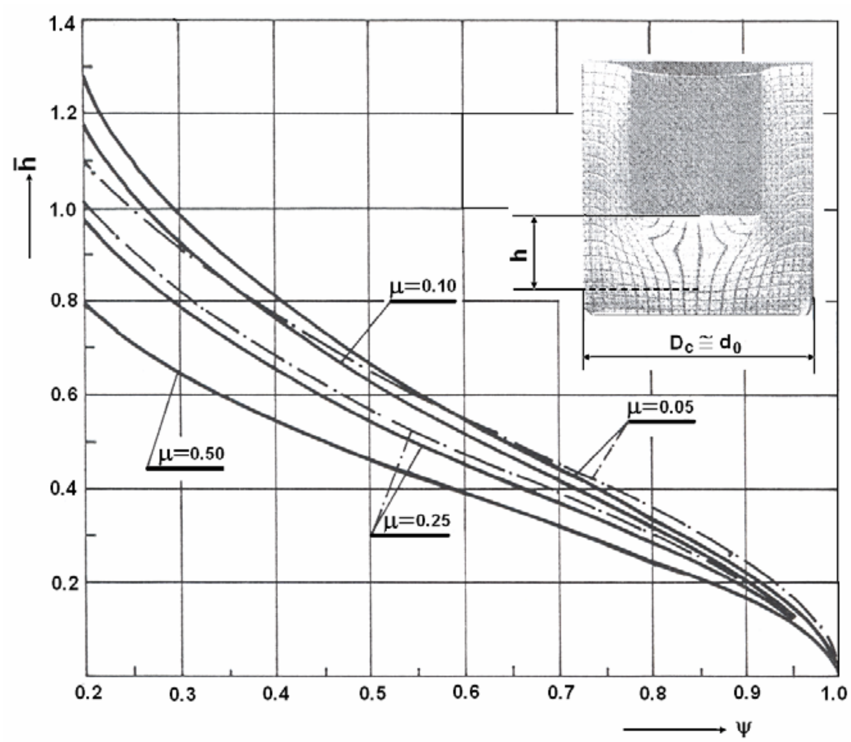


Figure 2 – Relative Height of ZPD

Since relation (15) is not suitable for direct application in practice, we can recommend a much simpler relation by Martirosyan [12]:

$$\overline{h} = \sqrt{\frac{(1-\psi)\sqrt{\psi}}{2\psi(2\mu+\sqrt{\psi})}} \tag{16}$$

Graph of function $\overline{h} = f(\psi)$, according to formula (16), is likewise given in Fig. 2 (denoted with full lines). From Fig. 2 we can observe very good agreement of values for $\overline{h}$, calculated by formulas (15) and (16).

3.1. Determination of Punch Pressure (Force)

The power of outer forces which is given by the punch is equal to the total dissipation power:

$$\dot{W}_o = p\,A_p V_0 \equiv \dot{W}_t \tag{17}$$

From relation (17) the average punch pressure (reduced force on the punch) is defined as a relation:

$$p = \frac{\dot{W}_t}{A_p V_0} \tag{18}$$

Direct application of formulas (14), (15) and (18) for calculation of the punch's reduced force is obviously unsuitable. That is why, on the basis of the given equations, and by taking into consideration relation (9) as well, we have constructed diagram $p/\overline{\sigma}_e = f(\mu,\psi)$ shown in Fig. 3. As can be seen in Fig. 3, the reduced extrusion pressure is considerably affected by the relative punch land (l_p / R_p).

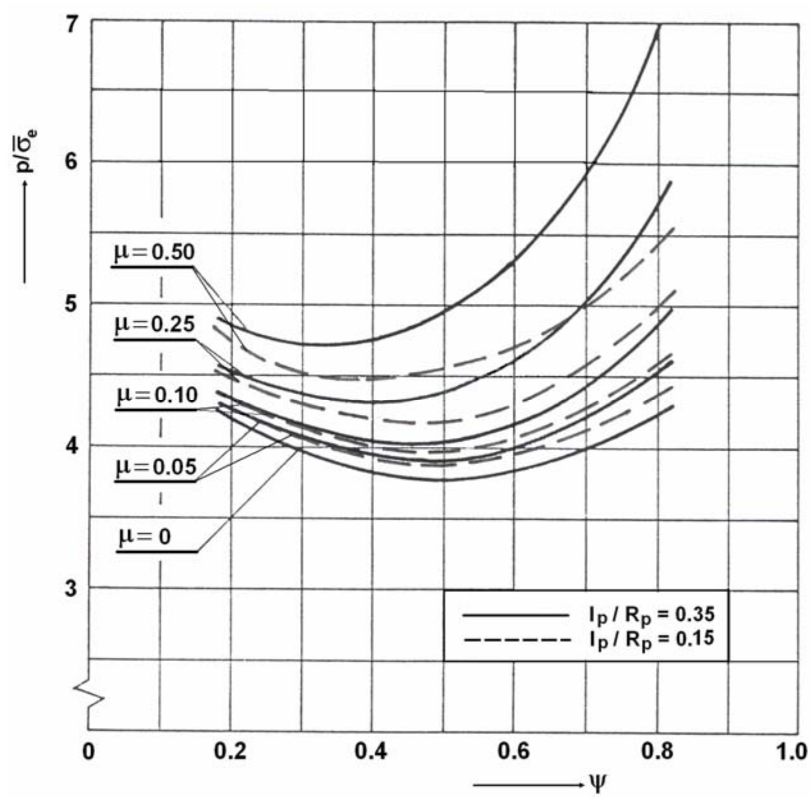


Figure 3 - Relative punch pressure in the process of backward cup extrusion (theoretical solution)

For ideal tribological conditions ($\mu = 0$) by the reological model of the rigid-plastic body, the respective curve shown in Fig. 3 is "symmetrical" with characteristic minimum (for $\psi = 0.5$). On the other curves in Fig. 3 we can notice a considerable increase of the relative punch pressure with friction coefficient increase μ and reduction ratio ψ . This phenomenon can be explained by the fact that the initial assumption in the theoretical analysis is that the contact friction stress is constant and proportional to flow stress which does not fully correspond to real forming conditions. Likewise, friction conditions are not identical at all friction surfaces which is experimentally confirmed. Accordingly, it is necessary to make adequate correction of the given theoretical results. Starting from the given analysis, as well as diagram in Fig. 3, relative extrusion pressure can be, in general, expressed in the following form:

$$\frac{p}{\bar{\sigma}_e} = F(\psi, \mu) \cdot \left(\frac{p}{\bar{\sigma}_e} \right)_{\mu=0} \tag{19}$$

In that sense, for calculating relative extrusion pressure, the approximate formula is suggested:

$$p / \bar{\sigma}_e \cong 5(1 + \mu\sqrt{\psi}) \left[1 - 0,95\psi (1 - \psi) \right] \tag{20}$$

If the thickness of the cup bottom b is less than height ZPD h (see Fig. 1), then, for calculating relative extrusion pressure at the end of extrusion process, in relation (14) $\bar{h}$ should be substituted by $\bar{b}$. Approximately the effect of the thin cup bottom on the magnitude of the extrusion pressure (force) can be encompassed by introducing the "conditional" logarithmic degree of deformation according to Shehter [15]:

$$\varphi = \ell n \left(\frac{D_c^2}{D_c^2 - D_i^2} \frac{h}{H_b} \right) \tag{21}$$

In doing so, height ZPD h can be determined by using formula (16).

3.2. Determination of average value of flow stress

The application of the rigid-plastic body model implies instantaneous transformation of the material into plastic state without hardening. Under real conditions of plastic forming such reological model can be simulated by forming some materials at high temperatures, when it can be assumed that $\sigma_e \approx \sigma_{e0} \cong const.$

Cold forming is characterized by intensive material hardening. The effect of material hardening on extrusion pressure, that is, force and deformation work, can be expressed by introducing into the calculation average value of flow stress $\bar{\sigma}_e$, as done in formula (20).

For materials with (approximately) linear flow curve $\sigma_e = f(\varphi_e)$, that can be given in either analytical or graphical form, average value of flow stress $\bar{\sigma}_e$ can be determined as arithmetic average of flow stress at the beginning σ_{e0} and at the end of forming process σ_{e1} :

$$\bar{\sigma}_e = \frac{1}{2}(\sigma_{e0} + \sigma_{e1}) \tag{22}$$

With materials which give intensive hardening, average value of flow stress $\bar{\sigma}_e$ can be determined by using the well-known mathematical expression about the average value of the function [3]:

$$\bar{\sigma}_e = \frac{1}{\varphi_1 - \varphi_0}\int_{\varphi_0}^{\varphi_1} \sigma_e d\varphi_e = \frac{1}{\varphi_1 - \varphi_0}\left(\int_0^{\varphi_1} \sigma_e d\varphi_e - \int_0^{\varphi_0} \sigma_e d\varphi_e\right) = \frac{w_{i1} - w_{i0}}{\varphi_1 - \varphi_0} \tag{23a}$$

where: w_{i0} - initial unit deformation work, w_{i1} - final unit deformation work.
Note: For practical reasons, further on in the paper we assume validity of identity $\varphi_e \equiv \varphi$.
In most cases it is $\varphi_0 = 0 (w_{i0} = 0)$, so that expression (23a) is reduced to a simpler form:

$$\bar{\sigma}_e = \frac{1}{\varphi_1}\int_0^{\varphi_1} \sigma_e d\varphi_e = \frac{w_{i1}}{\varphi_1} \tag{23b}$$

For most materials used for cold forming the yield curve can be represented in the form of power function $\sigma_e = C\varphi^n$, so that, according to formula (23b), it is obtained that:

$$\bar{\sigma}_e = \frac{C}{\varphi_1}\int_0^{\varphi_1} \varphi^n d\varphi = \frac{C\varphi_1^n}{1+n} = \frac{\sigma_{e1}}{1+n} \tag{23c}$$

In calculating average value of flow stress there is also the problem of defining representative indicator of deformation state – degree of deformation. For backward cup extrusion, in practice, an indicator of billet change of shape is taken to be the reduction ratio:

$$\psi = \frac{A_0 - A_1}{A_0} = \left(\frac{D_i}{D_0}\right)^2 \cong \left(\frac{D_p}{D_c}\right)^2 \tag{24}$$

where: A_0 - initial cross-section area, A_1 - final cross-section area.
For the needs of calculations and theoretical analysis, more suitable is the application of logarithmic degree of deformation:

$$\varphi = \ell n\frac{A_0}{A_1} = \ell n\frac{1}{1-\psi} \tag{25}$$

Many experiments show that there is remarkable non-homogeneity of local deformations within the ZPD and along cross-section of the extruded cup. Degree of deformation φ, defined by expression (25), represents only ''average'' value of local equivalent deformations by cross-section of the cup in ''steady'' state of extrusion process. That is why some authors express logarithmic degree of deformation in a different way. For instance, Siebel [1], [16], proposes the following relation:

$$\varphi = \ell n \frac{1}{1-\sqrt{\psi}} - 0,16 \tag{26}$$

According to other authors, average value of flow stress $\overline{\sigma}_e$ can be determined with respect to average value of deformation degree $\overline{\varphi}$. Thus, by Dipper's ''double upsetting'' model [1], [5], [16], average degree of deformation is defined in the form:

$$\overline{\varphi} = \ell n \frac{h_0}{H_b}\left[\psi + \left(1 - \frac{D_p}{8t}\right)(1-\psi)\right] = \ell n \frac{h_0}{H_b}\left[\psi + \left(1 + \frac{1}{4}\frac{\sqrt{\psi}}{1-\sqrt{\psi}}\right)(1-\psi)\right] \tag{27}$$

where: t - wall thickness of the cup.

Formula (26) gives somewhat enlarged values for logarithmic degree of deformation with respect to formula (25). On the other hand, a serious shortcoming of formula (27) lies in the fact that logarithmic degree of deformation practically depends only on relation l_p / H_b ($l_p / H_b \approx 4 \div 10$).

For further analysis we will use formulas (23), (24) and (25).

4. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

For qualitative and quantitative check-up of theoretical results given in this paper we can use descriptions of experimental research done by Kast [14]. Experimental results refer to steels 16MnCr5 and Ck 15 which are otherwise very often used in cold bulk forming processes. The chemical composition as well as mechanical properties of these steels are given in Tab. 1 and Tab.2. The workpieces in soft-annealed state are used. The chosen method of lubrication is the common one: zinc phosphate+MoS_2-powder.

Table 1 - Chemical composition of steels

Material	C	Si	Mn	P	S	N	Al	Cr
	(%)							
Ck 15	0.13	0.27	0.41	0.013	0.030	0.005	0.016	-
16MnCr 5	0.16	0.27	1.03	0.007	0.027	-	-	1.02

Table 2 - Mechanical properties of steels

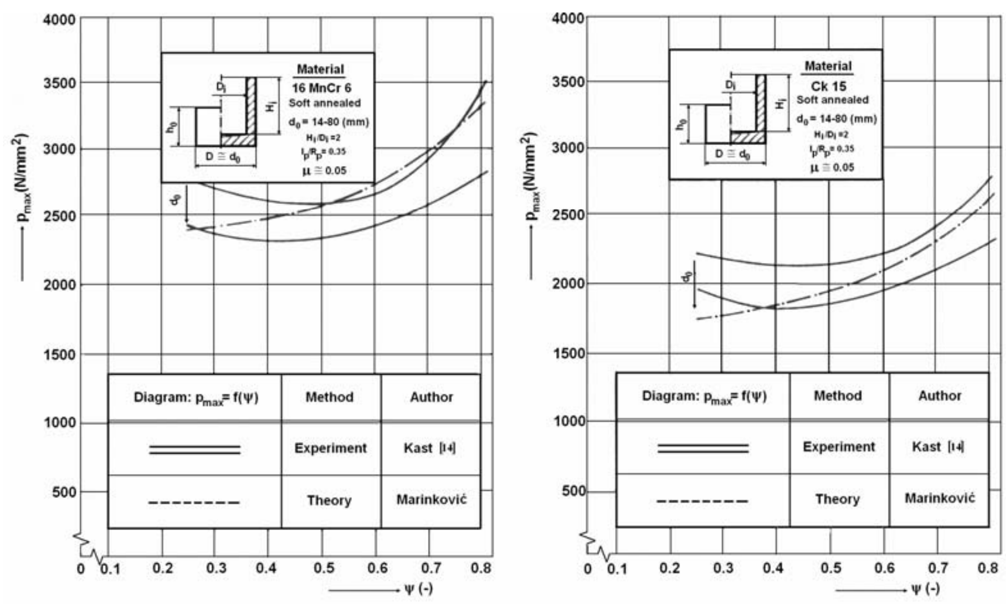
Material	HB	Rm	$R_{0.2}$	A_{10}	$\sigma_e = C \cdot \varphi_e^n$
	(N/mm^2)			(%)	(N/mm^2)
Ck 15	970 ÷1100	376	236	33.5	$642.7 \cdot \varphi^{0.209}$
16MnCr 5	1370 ÷1550	514	293	26.1	$802.7 \cdot \varphi^{0.156}$
Heat treatment: Soft-annealing 993 K (4h), cooling in furnace					

Comparative theoretical and experimental results are given in Fig. 4. Theoretical values for the extrusion pressure are obtained by using relations (25), (23c) and (20), with the assumption that fiction coefficient is $\mu = 0.05$.

It is well known that the process of cold backward cup extrusion of steel is realized in interval $0.25 < \psi \leq 0.85$ [1], [14]. In such a wide interval of reduction ratio there is good agreement between calculation and experimental results for extrusion pressure.

Theoretical results point to the tendency of the extrusion pressure drop with the decrease of reduction ratio. This can be explained by changed kinematics of material flow in the ZPD at small deformations ($\psi < 0.25$). Then the process of backward cup extrusion, by its kinematic scheme, transforms into cavity extrusion [13]. On the other hand,. Schmitt [16] has, on the basis of his experiments, concluded that increase of extrusion pressure with decrease of reduction ratio emerges only at relatively high workpieces ($h_0 / d_0 > 0.5$). This phenomenon is explained by changed friction conditions at the contact of the workpiece and the die during the punch's deep penetration into the material. Other authors have also found high gradiant of extrusion pressure increase at high values of relative deformation degree.

While analyzing deviations of theoretical from experimental results we should keep in mind the presence of errors in measurements as well as the impact of adopted approximations and simplification (especially in considering the phenomenon of material hardening) on calculation.



a) b)

Figure 4 – Comparative theoretical and experimental results for extrusion pressure according to approximate formula (20)

5. CONCLUSION

In all the aspects of its application, the backward cup extrusion process represents a critical operation because of extreme loading of the tool's working parts, namely, its variability during the forming process.
The paper presents a simplified cinematically admissible velocity field for analyzing the process of conventional backward cup extrusion. On the basis of the recommended field of velocities the average punch pressure (force) was determined by minimizing total power dissipation.
Since the solution of the upper bound is impossible to obtain in the closed form, the calculation was carried out numerically and shown in the paper in the form of diagram (by formulas (14) and (15)). To meet engineering needs, the approximate formula for determining extrusion pressure is derived. The theoretical extrusion pressures, calculated by the proposed approximate formula, were compared with the large experimental results and their satisfactory agreement is established.
The analysis carried out in this paper refers to the application of flat nose punch. The experiments have shown that the backward extrusion force can be lessened by applying the punch with conical nose. For practical reasons, instead of these, those to recommend are punches with their nose in the form of truncated cone. However, in this case the extrusion force reduction is negligible.

REFERENCES

[1] Spur G., Stöferle Th.: Handbuchder Fertigungstechnik, Band 2/2, Umformen, Carl Hanser Verlag, München/Wien, 1984.

[2] Kammerer M., Pöhlandt K., Tekkaya A.E..: Nono-conventional extrusion of less-common materials. J. Mater. Process. Technol., Vol. 49, 1995, pp. 345-354

[3] Jahnke H., Retzke R.,Weber W.: Umformen und Schneiden, Veb Verlag Technik, Berlin, 1972.

[4] Vilotić D., et al.: Application of net shape and near-net shape forming technologies in manufacturing of roller bearing components and cardan shafts. Journal for Technology of Plastcity, Vol. 32, 2007, pp. 87-104.

[5] Geiger R.: Der Stofffluß biem kombinierten Napffließpressen. Verlag W. Girardet, Essen, 1976.

[6] Lee D.J., Kim D.J., Kim B.M.: New processes to prevent a flow defect in the combined forward-beckward cold extrusion of a piston-pin. J. Mater. Process. Technol., Vol.138, 2003, pp. 422-427

[7] Marinković V.: Novi tehnološki postupak izrade šupljih elemenata sa vencem plastičnim deformisanjem u hladnom stanju ili toplom stanju. 1. međunarodni skup TM′93, , Vrnjačka Banja, 1993. s.392-397 (in Serbian)

[8] Bauschhausen A., Lee J.Y., Weinmann K., Altan T.: Evaluation of luibricant and friction in cold forging using a double backward extrusion process. J. Mater. Process. Technol. Vol. 33, 1992, pp. 95-108

[9] Schrader T., Shirgaokar M., Altan T.: A critical evaluation of the double cup extrusion test for selection of cold forging lubricants. J. Mater. Process. Technol. Vol.189, 2007, pp. 36-44.

[10] Kudo H.: Some Analytical and Experimental Studies of Axisymmetric Cold Forging and Extrusion-I. Int. J. Mech. Sci. 2-1/2, 1960, pp. 102-127.

[11] Tarnovskiy I.Ya.,Ganago O.A., Vaysburd R.A.: Deformacii i usiliya pri zakrytoj proshivke. Chernaya Metallurgiya, No 4, 1960. s. 99-108 (in Russian)

[12] Martirosyan F.A.: Opredelenie polyaa napryazheniya i razmerov ochaga plasticheskih deformacii pri obratnom vydavlivanii. Izv.Vuz. Mashinostroenie, No 10, 1966, s.147-154. (in Russian)
[13] Storozhev M. V., Popov E. A.: Teoriya obrabotki metallov davleniem. Mashinostroenie, Moscow, 1977. (in Russian).
[14] Kast D.: Modellgesetzmäßigkeiten beim Rückwärtsfließpressen geometrisch ähnlicher Näpfe. Verlag W. Girardet, Essen, 1969.
[15] Ovchinnikov A.G.: Osnovy teorii shtampovki vydavlivaniem na pressah, Mashinostroenie, Moskow, 1983. (in Russian)
[16] Schmitt G.: Untersuchungen über das Rückwärts-Napffließpressen von Stahl bei Raum-temperatur. Verlag W. Girardet, Essen, 1968.
[17] Plančak M.: Naponsko-deformaciono stanje u procesima hladnog istiskivanja čelika. Doktor. disertacija, Fakultet tehničkih nauka, Novi Sad, 1984. (in Serbian)
[18] Thomsen E.G., Yang C.T., Kobayashi S.: Mechanics of Plastic Deformation in Metal Processing. Macmillan Company, New York, 1965.
[19] Johnson W., Mellor P.B.: Engineering Plasticity, Van Nostrand Reinhold Company, London, 1973.
[20] Kolmogorov V.L.: Mehanika obrabotki metallov davleniem, Metallurgiya, Moskow, 1986. (in Russian)
[21] Marinković V.: Deformaciono ojačavanje materijala pri obradi deformisanjem u hladnom i toplom stanju, Mašinski fakultet, Niš, 1995. (in Serbian)
[21] Bramley A.N. Computer aided forging Design. Ann. CIRP, Vol 36, 1987, pp. 135-138.
[22] Osman F.H., Bramley A.N., Ghobrial M.T.: Forging and preform design using UBET. Proc. 1st ICTP,1984, 1984, pp. 563-568.

ODREĐIVANJE RADNOG PRITISKA PROCESA SUPROTNOSMERNOG ISTISKIVANJA ŠUPLJIH PROFILA

Velibor Marinković

Univerzitet u Nišu, Mašinski fakultet, Niš, Srbija

REZIME

Proces suprotnosmernog istiskivanja šupljih profila se primenjuje za izradu širokog spektra delova u različitim granama industrije, bilo kao pojedinačna operacija, bilo kao jedna sekvenca u višestupnjevitom tehnološkom procesu, a naročito efektivno u različitim procesima kombinovanog istiskivanja.

U svim vidovima primene proces suprotnosmernog istiskivanja šupljih profila predstavlja kritičnu operaciju, zbog ekstremnog opterećenja radnih delova alata, koje je promenljivo tokom procesa deformisanja. Ekstremno opterećenje alata, naročito tiskača, predstavlja ponekad ograničavajući faktor primene procesa suprotnosmernog istiskivanja šupljih profila. Ukoliko tiskač nije pravilno centriran i vođen, svaki poremećaj može dovesti do nestabilnosti procesa istiskivanja i oštećenja ili loma tiskača. Zato je proračun radnog pritiska (sile) tiskača od prvorazrednog značaja.

Za proračunradnog pritiska tiskača u radu je primenjen približan energetki metod, poznat pod nazivom metod gornje granice. Radi lakše primene u inženjerskoj praksi izvedena formula je prikazana u vidu dijagrama. S druge strane, za potrebe projektovanja i proračuna tehnoloških procesa deformisanja pomoću računara, u radu je predložena i približna formula. Utvrđeno je da se odstupanje teorijskih i eksperimentalnih rezultata nalazi u granicama prihvatljive greške.

Ključne reči*: Hladno suprotnosmerno istiskivanje, Pritisak istiskivanja, Metoda gornje granice*