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Using the Clearing Function to evaluate the Pricing Capacity of Resources at Low Levels of Utilization in Production Planning

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Abstract

It is well known from the daily industrial experience that high levels of throughput in production depend on high levels of work–in–process or releases into the system, and that high levels of work–in–process may increase the total lead time into the systems, decreasing expected revenues. This clearly suggests that sometimes increasing production capacity is in our best interest even before it becomes tight, even though the necessary information for accomplishing this is not provided by most of the approaches used for these issues, and in particular, by classical linear programming models. Recently some authors developed a framework to circumvent this drawback in the approach of linear programming based on the concept of clearing function that strives, together with the approach of linear programming, to allow for the pricing of low levels of capacity utilization. Nevertheless, the resulting new model was not treated directly but approximated by a linear model, which received a classical treatment, revealing very little news. In this paper we treat this new model directly, and furthermore, we took a new approach for the linear model, which in our view, has produced a new and deeper vision for the subject.

Key words: *Clearing Function, Convex Programming, Utilization of Capacity, Work-In-Process.*

1. INTRODUCTION

It is well known from the daily industrial experience that high levels of throughput in production depend on high levels of work–in–process (WIP) and releases into the system, and that high levels of WIP can increase the total lead time in the systems, decreasing expected revenues [12], [4]. This clearly suggests that increasing production capacity is in our best interest even before it becomes tight. However, the necessary information about when and how to accomplish this is not provided by the most commonly used approaches to these issues which are the classical Linear Programming (LP) models. In fact, as longer production planning is modelled using LP, WIP is not explicitly considered, it is just a consequent balance among levels of production, final good inventory (FGI), and demand. Moreover, throughput is directly related to capacity, ignoring other factors which contradict both the practical experience coming from flow shop as well as the insights coming from queuing theory [3]. Therefore the use of LP to model production planning must be modified to incorporate the appropriate frameworks to deal with the issues of WIP and thus allow for the pricing of low levels of capacity utilization. It is important to clarify that the term capacity used in this paper stands for nominal capacity in most instances; however, we do not strictly follow the recommendations from Elmaghraby [10]. The framework we develop here to modify the classical approach of LP is based on the concept of clearing function (CF) first suggested by Graves [9], Karmarkar [11], and Srinivasan *et al.* [2], and more recently extensively used by Asmundsson *et al.* [3], Ali *et al.* [1], Irdem *et al.* [6], and Missbouer and Uzsoy [7], in the same setting considered here, just to name a few recent papers.

The approach we use in this work is twofold: on the one hand we assume that even though the behaviour of CF function is known, in general there is no analytical formulation at hand to be used and only numerical information about the medium number of throughput for any level of WIP is available. This assumption is mainly supported by the queue theory and results from several different numerical experiments that show a great deal of variability in throughput in response to the same level of WIP, which suggests that quite different analytic models could be used to represent the throughput as a function of a measure of WIP, as long as they observe that the data distribution has clearly standard concave behaviour. In this case the CF function could be approximated using a regression concave curve or a piecewise affine approximation according to its concave behaviour, i.e., the angular parameters decrease from nominal production value (or from 1), to zero, and the linear parameters increase from zero (or from a small value) to nominal production capacity. The results for pricing capacity that come from the approach of piecewise affine approximation agrees in general with the results forecasted in the queue theory already available in the literature, and thus we do not go any farther about it, however, we present a sensitivity analysis for the piecewise affine approximation from inner to outer approximation. The second aspect is when we can manage to have an analytical formulation for the CF function. In this case we price low levels of capacity throughout a convex programming model, and then we perform a sensitivity analysis for the model related to the CF function using perturbation on the convex model.

We start this work by discussing LP models for pricing capacity at low levels of utilization in an environment of single–stage single production–inventory system, and then extend the model to single and multi-stage multi–product systems. The approach we used complement the linear outer–approximation approach for CF function used in Ali *et al.* [1], introducing a linear inner–approximation approach, and directly considering the convex approximation approach, but, we do not use any specific analytical expression for the CF function. It is worthy to note that combining inner and outer–approximation to the CF function unveils several aspects of pricing capacity not clearly seen up until now, as well as allowing for an interesting sensitivity analysis related to CF function by means of variations from inner to outer piecewise affine approximation to the CF function. These results compare with those generated by the sensitivity analysis for the convex model, which suggest that the choice of the model, whether strictly convex or convex piecewise linear, depends mainly on which kind of information is available about the CF function.

To summarize, there are two main results presented in this work. One is how the modified production planning LP model gives answers to small changes in the CF function which allow for the pricing of low levels of capacity utilization in different periods of the planning horizon, and second, how the convex approximate model responds to small perturbations of the model. The remainder of this paper is organized as follows: In Section 2 the working model is built upon the classic LP model and the main assumptions regarding the model and CF function are discussed as well. In Section 3, a small series of numerical experiments are resented to illustrate the theory and final conclusions are presented.

2. FORMULATION OF THE LP MODEL

The approach used here for modeling production using the role of CF function follows the path of Ali *et al.* [1], Missbouer and Uzsoy [7], Irdem *et al.* [6], which in turn follows some previous works from Graves [9] and Karmarkar [11]. However, here we use a rather different assumption which is that we only have access to numerical information about the relationship between levels of WIP and throughput.

In the classical LP approach, capacity is supposed to be instantaneously available and kept constant throughout each period, and each period may be stretched long enough to avoid any violation of capacity. It is well known that the LP approach always prescribes that dual variables associated to capacity will remain at zero as long as capacity is not fully utilized. However, practical experience has shown that WIP and costs associated with lead time are significant aspects of the production cost even when the levels of capacity utilization are below their upper limits. Although bounded and proportional models take these aspects into consideration they are clearly poor approximations for the CF function, and it was only recently with the works of Ali *et al.* [1], Irdem *et al.* [6], and Missbouer and Uzsoy [7], that this started being revised, to be used for the pricing of lower levels of capacity utilization. Their approach proposes to approximate the CF concave function by means of a bundle of piecewise affine functions, and so they model the problem as a modified LP problem, thus keeping the simplicity of LP models while allowing for the possibility of pricing low levels of capacity utilization using the standard LP sensitivity analysis. However, in order to price low levels of capacity they had to circumvent some aspects related to complementary slackness, and the way they found to modify the CF function approach was to introduce a new variable associated with a new equality constraint, in order to force the possible existence of a positive dual variable. Since this new variable was introduced as a decision variable, and the variation of the CF function was related to the inverse variation of this new variable, the role of the CF function may have been misinterpreted and then masked by the piecewise linear approximation for the CF function. However, the approach had some interesting consequences; a practical way to approximate the CF function using a bundle of piecewise affine functions, as well as associating pricing capacity to functional constraint instead of bounded decision variables, which may avoid degenerate solutions, and consequently non trivial duality analysis.

The piecewise affine functions used in most of the literature to represent the CF function was intended to be an outer–approximation for a regression CF function, and in this sense it generally overestimates the CF function it represents. However, if the number of affine functions in the bundle increase arbitrarily this approach eventually converges to the solution predicted by the regression concave CF function itself [8], which means that here the size of the bundle matters.

In this work we complement the aforementioned approach of piecewise affine function introducing an inner approach, and then exploring the variability from the inner to outer approximation to CF function as a way of analysing the sensitivity of the modified LP model, as well as a direct approach to the convex model for the problem, including a sensitivity analysis based on the perturbation of the convex model.

2.1The Classic LP Model

Let's start with the problem addressed by Karmarkar [11], for a single product produced in a single machine, defining the following set of parameters and decision variables,

- c_t - unit cost of product at period t;
- h_t - unit cost of handling inventory at period t;
- C_t - Available capacity at beginning of period t;
- D_t - Demand to be satisfied by the end of period t;
- X_t - Level of production at the end of period t;
- I_t - Level of inventory at the end of period t.

The classic LP production planning model is generally formulated as,

$$Minimize \sum_{t=1}^{T}\{c_t X_t + h_t I_t\}$$

$$s.t. \quad I_t = I_{t-1} + X_t - D_t, \quad \forall t, \tag{1}$$

$$X_t \le C_t, \quad \forall t,$$

$$X_t, \quad I_t \ge 0 \quad \forall t,$$

and then if inventory in any period t is written as a function of production X_t, demand D_t, and initial inventory I_0 as,

$$I_t = I_0 + \sum_{i=1}^{t} X_i - \sum_{i=1}^{t} D_i, \; t = 1, \dots, T, \tag{2}$$

then the LP model becomes expressed only in terms of final good inventory as,

$$Minimize \sum_{t=1}^{T}\{c_t + \sum_{i=1}^{T} h_i\} X_t$$

$$s.t. \quad I_t = I_0 + \sum_{i=1}^{t} X_i - \sum_{i=1}^{t} D_i, \quad \forall t, \tag{3}$$

$$X_t \le C_t, \quad \forall t,$$

$$X_t \ge 0, \quad I_t \ge 0 \quad \forall t,$$

2.2 Reformulation of the Classic LP Model

Problem (3) does not make explicitly any reference to WIP (W_t) and does not include releases (R_t) at the beginning of any period t as a decision variable, which makes the model unsuitable for pricing low levels of capacity utilization. However, we may modify this LP model to incorporate WIP and releases, using the following direct extension of model (3),

$$Minimize \sum_{t=1}^{T}\{c_t X_t + h_t I_t + w_t W_t + r_t R_t\} \tag{4}$$

$$s.t. \quad I_t = I_0 + \sum_{i=1}^{t} X_i - \sum_{i=1}^{t} D_i, \quad \forall t, W_t = -X_t + W_{t-1} + R_t, \quad \forall t,$$

$$X_t \le C_t, \quad \forall t,$$

$$X_t \ge 0, \quad I_t \ge 0, W_t \ge 0, R_t \ge 0, \quad \forall t,$$

which explicitly uses WIP and releases as decision variables. The new parameters r_t and w_t denote the unit cost of releases and WIP holding, respectively, and the new decision variables W_t and R_t denote respectively the amount of products in WIP and the releases at the beginning of the period t. However, this model does not recognize the impact of the workload of production resources on the lead times of the system, which is considered partially only up on the inclusion of a CF function that governs production based on the workload of the system. To define a CF function, let $\Psi_t(W_{t-1}, R_t)$ be the concave CF function which governs production as a function of the workload of the system, and links expected throughput, X_t, with levels of WIP and releases in each period t, thus implicitly bringing lead time to the scenario. Then the convex model that results from the modified LP model is,

$$Minimize \sum_{t=1}^{T}\{c_t X_t + h_t I_t + w_t W_t + r_t R_t\} \tag{5}$$

$$s.t. \quad I_t = I_0 + \sum_{i=1}^{t} X_i - \sum_{i=1}^{t} D_i, \quad \forall t, W_t = -X_t + W_{t-1} + R_t, \quad \forall t,$$

$$X_t - \Psi_t(W_{t-1}, R_t) \le 0, \quad \forall t, X_t \ge 0, \quad I_t \ge 0, W_t \ge 0, R_t \ge 0, \quad \forall t,$$

If we assume that W_0=0, then from

$$W_t = -X_t + W_{t-1} + R_t, \; t = 1, \dots, T,$$

$$W_t = \sum_{i=1}^{t} R_i - \sum_{i=1}^{t} X_i = \sum_{i=1}^{t} (R_i - X_i), \tag{6}$$

Since W_T=0 for any optimal solution, then, for $t = 1, \dots, T$,

$$0 = W_T = \sum_{i=1}^{T} R_i - \sum_{i=1}^{T} X_i = \sum_{i=1}^{T} (R_i - X_i), \tag{7}$$

which requires that releases must be transformed into production by the end of the planning horizon. This clearly suggests that CF function can be written uniquely as a function of releases in the planning horizon, nevertheless, this will not be discussed here. In this linear approach, for each period t, the CF function $\Psi_t(W_{t-1}, R_t)$, is approximated by a piecewise linear function. The approximation is built in the following way: Let a_{it} and b_{it} be information provided by the CF function, with $0 < a_{it}$ and $0 < b_{it} < C_t$, where a_{it} gives information about the speed of variation of throughput, and b_{it} gives information about the level of throughput related to nominal production capacity, then the CF function $\Psi_t : R \to R$ can be approximate by $\Phi_t(\overline{W}_t) = minimum_i \{a_{it}\overline{W}_t + b_{it}\}$ to any degree of required accuracy [8], and henceforth used. Thus, the modified LP model becomes,

$$Minimize \sum_{t=1}^{T}\{c_t X_t + h_t I_t + w_t W_t + r_t R_t\}$$

$$s.t. \quad I_t = X_t + I_{t-1} - D_t, \quad \forall t,$$

$$W_t = -X_t + W_{t-1} + R_t, \quad \forall t, \tag{8}$$

$$X_t - a_{it} W_{t-1} - a_{it} R_t \le b_{it}, \quad \forall t,$$

$X_t \geq 0, \quad I_t \geq 0, W_t \geq 0, R_t \geq 0, \quad \forall t,$

According to this formulation, WIP and release must accomplish the overall optimality condition. Moreover, the model has no box constraints at all and unlike the classic LP model it is not necessarily solution degenerate for pricing capacity utilization.

2.3 Pricing low levels of capacity

The mathematical model defined in (8) allows periods of congestion, non–congestion, and idle periods for the production plan, however we could impose initial conditions on I_t, W_t and R_t, in order to drive the model for a specific state. For instance, the dual LP model for the primal model (8) for $I_0 = I_T = W_0 = W_T = 0$, is written as,

$$Maximize \sum_{t=1}^{T} D_t\alpha_t - \sum_{t=1}^{T}\sum_{i=1}^{N} b_{it}\gamma_{it}$$

$$s.t. \quad \alpha_t - \beta_t + \sum_{i=1}^{N}\gamma_{it} \leq c_t, \quad \forall t,$$

$$-\alpha_t + \alpha_{t+1} \leq h_t, \quad \forall t,$$

$$-\beta_t + \beta_{t+1} - \sum_{i=1}^{N} a_{it}\gamma_{it} \leq w_t, \quad \forall t,$$

$$\beta_t + \sum_{i=1}^{N} a_{it}\gamma_{it} \leq r_t, \ \forall t, \tag{9}$$

$$\alpha_t \geq 0, \beta_t \geq 0, \quad \gamma_{it} \geq 0, \quad \forall t,$$

$$i = 1, \dots, N, t = 1, \dots, T.$$

Where α_t is the associated price to inventory at period t, β_t is the price associated to WIP at period t, and $\gamma_t = \sum_{i=1}^{N}\gamma_{it}$ is the price associated to levels of capacity utilization at period t. The objective function of this problem has the interesting aspect of suggesting that at optimality, any cost paid to increase capacity is offset by production gains, since it must satisfy the optimality condition, i.e., the dual objective function must remain constant and equal to the optimal primal value. In the same way, if demand increases so do the prices associated with utilization, all meaning to say that the pricing of utilization starts increasing long before the capacity is exhausted.

The total price p_t associated to capacity at any period t is given by,

$$p_t = \sum_{t=1}^{T}\sum_{i=1}^{N} b_{it}\gamma_{it}, \tag{10}$$

And for different small variations of the parameter b_{it} which stands for small perturbations of the affine function from the piecewise affine approximation of the CF function, we define the perturbations of the CF function as a variation from

$$a_{it}W_{t-1} + a_{it}R_t + b_{it} = 0, \tag{11}$$

which stands for a basic component from the bundle approximation to the CF function, to

$$a_{it}W_{t-1} + a_{it}R_t + b_{it} + \eta_{it} \in \mathbb{R} = 0, \tag{12}$$

η_{it} defines the range of variation from inner–approximation to outer–approximation for the CF function, depending on its real value in $\mathbb{R}$. Then, if we define the range of variability of our CF function by Δp_t, the variability of the associated prices will be,

$$\Delta p_t = \sum_{t=1}^{T}\sum_{i=1}^{N}\eta_{it}\gamma_{it}, \tag{13}$$

If $\eta_{it} \geq 0$, Δp_t is the maximum price we are willing to pay for increasing capacity to cope with any increase in demand, or just to improve the overall capacity of the system. We are not going any further with this because this issue is already well covered in the aforementioned literature [1], [2], [3], [4], [6], [7]. Now we turn to model (5), which is the basic convex programming model for the CF model.

2.4 Perturbation of the CF Convex Model

Now let's go back to model (5), which is a convex programming problem, and for the sake of simplicity let's have it generally written as,

$$Minimize\ f_0(x)$$

$$s.t. \quad f_t(x) \leq 0, \ \forall t \tag{14}$$

$$h_t(x) = 0, \ \forall t$$

where the constraints of model (5), $-X_t \leq 0$, $-I_t \leq 0$, $-W_t \leq 0,\ and\ -R_t \leq 0$, were incorporated into the inequalities constraints $f_0(x) = c_tX_t + h_tI_t + w_tW_t + r_tR_t$, is the objective linear function, $f_t(x) = X_t - \Psi_t(\widetilde{W}_t)$, is the strictly convex function, and

$$h_t(x) = \begin{pmatrix} X_t + I_{t-1} - D_t - I_t \\ X_t + W_{t-1} - R_t - W_t \end{pmatrix} \tag{15}$$

are the two sets of affine functions. There is a practical reason for proceeding in this manner since the specific formulae for the clearing function Ψ_t, depends on some particular characteristics of the system, even though in any case, $-\Psi_t$ is a strictly convex function. Let's assume that problem (14) has a feasible solution which satisfies the Slater condition, which assumes that the relative interior of the feasible solution set is non-empty. The Lagrangian of this problem is,

$$l(x,\lambda,\mu) = f_0(x) + \sum_t \lambda_t f_t(x) + \sum_t \mu_t h_t(x), \tag{16}$$

$$\lambda \geq 0,$$

and its dual function is

$$g(\lambda,\mu) = \underset{x}{infimum}\ l(x,\lambda,\mu),$$

$$\lambda \geq 0, \tag{17}$$

which is always a non-smooth concave function since it is a piecewise infimum of a family of affine functions. If x is a feasible solution for (14), then,

$$g(\lambda,\mu) \leq f_0(x), \ \forall(\lambda,\mu), \lambda \geq 0. \tag{18}$$

This important property, called weak duality, is easily verified since for all feasible solutions for (14) we have that,

$$\sum_t \lambda_t f_t(x) + \sum_t \mu_t h_t(x) \leq 0. \tag{19}$$

Moreover, if x^* is an optimal solution for (14), and (λ^*, μ^*) an optimal solution for the so called dual problem,

$$Maximize\ g(\lambda, \mu)$$
$$s.t.\quad \lambda \geq 0, \tag{20}$$

then, since the Slater condition is satisfied,

$$g(\lambda^*, \mu^*) = f_0(x^*). \tag{21}$$

Problem (20) is referred to as a dual problem for problem (14), and its optimal value $g(\lambda^*, \mu^*)$ gives the best lower bound for problem (14), while the optimal value of primal problem, $f_0(x^*)$, gives the best upper bound for the dual problem (20). Now let's consider small perturbations (u_t, v_t) for problem (14), which give us,

$$Minimize\ f_0(x)$$
$$s.t.\quad f_t(x) \leq u_t,\ \forall t \tag{22}$$
$$h_t(x) = v_t,\ \forall t$$

where u_t can be either positive, in which case the constrained f_t is relaxed, or negative, in which case the constraint f_t is tightened. According to model (5), the perturbation u_t can be interpreted as variations of the CF function, and since strong duality is attained, this allows us to have valuable information on how the value of an optimal solution to model (14) gives answers to the perturbations of the CF function. Likewise, changes on the right hand side of equality constraints can be interpreted as variations over boundary conditions about demand, releases, and WIP, even though it doesn't matter too much here since we are mainly interested in how model (14) gives responses to small perturbations of the CF function. The Lagrangian function of the perturbed problem (22) is,

$$L(x, \lambda, \mu) = f_0(x) + \sum_t \lambda_t [f_t(x) - u_t] + \tag{23}$$
$$\sum_t \mu_t [h_t(x) - v_t], \qquad \lambda \geq 0,$$

and its dual function is $G(\lambda, \mu) = inf_x L(x, \lambda, \mu), \lambda \geq 0$. Let's define a set G of the optimal solutions for problem (22), for each perturbation (u_t, v_t), and a function G^* over, defined as $G^*(u_t, v_t)$. Observe that

$$G^*(0,0) = g(\lambda^*, \mu^*) = f_0(x^*), \tag{24}$$

and from strong duality, for any pair $(x^*, (\lambda^*, \mu^*))$ of optimal solution,

$$g(\lambda^*, \mu^*) \leq f_0(x^*) + \sum_t \lambda_t^* f_t(x^*) + \sum_t \mu_t^* h_t(x^*)$$
$$\leq f_0(x^*) + \sum_t \lambda_t^* u_t(x^*) + \sum_t \mu_t^* v_t(x^*) \tag{25}$$
$$= f_0(x^*) + \lambda^{*^T} u + \mu_t^{*^T} v\ ,$$

which implies by concavity of $G^*(u, v)$, and the subgradient relation $(-\lambda^*, -\mu^*) \in \partial G^*(0,0)$, that $\forall (u, v)$,

$$G^*(u, v) \geq G^*(0,0) - \lambda^{*^T} u - \mu_t^{*^T} v\ . \tag{26}$$

To compare with the above linear approach, we have that for all periods t,

$$\lambda_t^* \leftarrow \sum_{i=1}^{N} \gamma_{it}^*, \tag{27}$$

where λ_t^* is the Lagrange multiplier for the convex model, and γ_{it}^* is the Lagrange multiplier associated to the piecewise affine approximation. From inequality (26) we can directly have several insights about the effect of perturbation on the CF function on the model, no matter whether Ψ_t is a strictly concave function or its piecewise linear approximation, since this insight comes from the general theory of convex duality. Some of the most obvious insights are listed below as a matter of explicitness:

1. if $u_t < 0$ is large and the t^{th} inequality is tight, then $G^*(u, 0)$ is guaranteed to greatly increase, depending on λ_t, which leads us to say that reducing capacity increases costs;
2. if $u_t < 0$ is small and the t^{th} inequality is tight, then $G^*(u, 0)$ may not increase too much, i.e., the production costs may or may not increase, depending on λ_t;
3. if $u_t > 0$ is large and the t^{th} inequality is tight, then $G^*(u, 0)$ may decrease too much, depending on λ_t;
4. if $u_t > 0$ and the t^{th} inequality is tight, then $G^*(u, 0)$ may not decrease too much, depending on λ_t, however, it can't increase production costs.

All this leads us to say that sometimes increasing capacity is the right way to provide conditions for lowering production costs. We must bear in mind that $G^*(u, v)$ is the value of the dual function for the perturbed problem, which gives an inferior bound for the value of the primal function for each perturbation pair (u, v), and hence, if it increases, so does the primal function, which means in the end, that production costs increase. And, on the contrary, if $G^*(u, v)$ decreases, so does the production costs. It is worthy to note that this technology provides a sensitivity analysis for all possible cases, regardless of whether or not the CF function is a strictly concave function or just a concave function given by the infimum of piecewise affine functions. The specific sensitivity analysis for the piecewise affine approximation is trivial from using the variability of the CF piecewise linear approximation η_t as a perturbation of the CF function, $\eta_t = u_t$, which leads us to the same conclusions pointed out above related to the perturbed convex model.

2.5 Multiple Resources and the Multiple Product Model

Now let's focus on the production capacity over the available amounts of different resources for each period in the planning horizon, and assume again that capacity is governed by the CF function over WIP for several different products. Since we begin to model the problem for a single resource, the next step is to extend the achievement to multiple resources. A direct extension from model (5) can be formulated as,

$$Minimize \sum_{t=1}^{T}\sum_{i=1}^{N}\{c_{it}X_{it}+h_{it}I_{it}+w_{it}W_{it}+r_{it}R_{it}\} \quad s.t. \quad I_{it}=X_{it}+I_{it-1}-D_{it}, \quad \forall i,\forall t,$$

$$W_{it} = -X_{it} + W_{it-1} + R_{it}, \forall i, \forall t, \tag{28}$$

$$\sum_{i=1}^{N} \xi_{it}X_{it} - \Psi_t(\sum_{i=1}^{N}\theta_{it}\overline{W_{it}}) \leq 0, \forall t$$

$$X_{it} \geq 0, I_{it} \geq 0, W_{it} \geq 0, R_{it} \geq 0, \forall i, \forall t,$$

where ξ_{it} is the consumption of resources necessary to produce one unit of product i at period t and θ_{it} relates to how much WIP we should have to meet the required demand for the same period. Despite the fact that this model looks quite reasonable at first glance it carries a strong drawback, since it clearly may create the capacity for a single specific product providing WIP for another. A good example of this can be found in Missbouer and Uzsoy [7]. Therefore, we must introduce a rule to prevent this undesirable behaviour in the model by requiring the capacity for each single product, and to do so, let's assume that the total consumption of resources for each product i is bounded by a fraction of total capacity, i.e., for all i and for all t,

$$\xi_{it}X_{it} \leq \rho_{it}\Psi_t(\textstyle\sum_{i=1}^{N}\theta_{it}\overline{W_{it}}), \ \rho_{it} \geq 0, \tag{29}$$

$$\sum_{i=1}^{N}\rho_{it} = 1$$

It is important to understand that ρ_{it} is a technical parameter that must be entered into to the model and not a new decision variable. Upon these assumptions, model (28) will become,

$$Minimize \sum_{t=1}^{T}\sum_{i=1}^{N}\{c_{it}X_{it}+h_{it}I_{it}+w_{it}W_{it}+r_{it}R_{it}\} \quad s.t. \quad I_{it}=X_{it}+I_{it-1}-D_{it}, \quad \forall i,\forall t,$$

$$W_{it} = -X_{it} + W_{it-1} + R_{it}, \forall i, \forall t, \tag{30}$$

$$\sum_{i=1}^{N} \xi_{it}X_{it} - \Psi_t(\sum_{i=1}^{N}\theta_{it}\overline{W_{it}}) \leq 0, \quad \forall t$$

$$\xi_{it}X_{it} - \rho_{it}\Psi_t(\sum_{i=1}^{N}\theta_{it}\overline{W_{it}}) \leq 0, \quad \forall t$$

$$X_{it} \geq 0, I_{it} \geq 0, W_{it} \geq 0, R_{it} \geq 0, \forall i, \forall t.$$

This model is still not well defined, since by concavity of Ψ_t, for all solutions satisfying the set of constraints (29) it will also satisfy,

$$\textstyle\sum_{i=1}^{N}\xi_{it}X_{it} - \Psi_t(\sum_{i=1}^{N}\theta_{it}\overline{W_{it}}) \leq 0, \tag{31}$$

thus, implying that the set of constraints (30) is redundant. Cleaning up the redundancies from the model, it becomes,

$$Minimize \sum_{t=1}^{T}\sum_{i=1}^{N}\{c_{it}X_{it}+h_{it}I_{it}+w_{it}W_{it}+r_{it}R_{it}\} \quad s.t. \quad I_{it}=X_{it}+I_{it-1}-D_{it}, \quad \forall i,\forall t,$$

$$W_{it} = -X_{it} + W_{it-1} + R_{it}, \forall i, \forall t, \tag{32}$$

$$\xi_{it}X_{it} - \rho_{it}\Psi_t(\sum_{i=1}^{N}\theta_{it}\overline{W_{it}}) \leq 0, \quad \forall t$$

$$X_{it} \geq 0, I_{it} \geq 0, W_{it} \geq 0, R_{it} \geq 0, \forall i, \forall t,$$

This model is likely more general than some previous related models appearing in the literature andcarries the suitable aspect where for the case, $\theta_{it} = \xi_{it} = \rho_{it} = 1$, it reproduces the initial model (5) for one single product and one single machine. Note that the equality $\theta_{it} = \xi_{it}$, states that all the resources will be transformed into production, and that of $\rho_{it} = 1$ just means that we have a unique resource. Some particular choices for these parameters, mainly those which take one parameter as a function of another must be done very carefully, taking into consideration the resulting composite CF function. Finally, adding up conditions (29), $\xi_{it}X_{it} \geq 0$, and $\theta_{it} \geq 0$, we ultimately generate the convex model,

$$Minimize \sum_{t=1}^{T}\sum_{i=1}^{N}\{c_{it}X_{it}+h_{it}I_{it}+w_{it}W_{it}+r_{it}R_{it}\} \quad s.t. \quad I_{it}=X_{it}+I_{it-1}-D_{it}, \quad \forall i,\forall t,$$

$$W_{it} = -X_{it} + W_{it-1} + R_{it}, \quad \forall i, \forall t, \tag{33}$$

$$\xi_{it}X_{it} - \rho_{it}\Psi_t(\sum_{i=1}^{N}\theta_{it}\overline{W_{it}}) \leq 0, \quad \forall i, \forall t$$

$$X_{it} \geq 0, \quad I_{it} \geq 0, W_{it} \geq 0, R_{it} \geq 0, \quad \forall i, \forall t,$$

The model for multiple resources and multiple products become a natural extension of model (33), and might be formulated as,

$$Minimize \sum_{t=1}^{T}\sum_{i=1}^{N}\{c_{it}X_{it}+h_{it}I_{it}+w_{it}W_{it}+r_{it}R_{it}\} \quad s.t. \quad I_{it}=X_{it}+I_{it-1}-D_{it}, \quad \forall i,\forall t,$$

$$W_{it} = -X_{it} + W_{it-1} + R_{it}, \forall i, \forall t, \tag{34}$$

$$\xi_{it}{}^{j}X_{it} - \rho_{it}{}^{j}\Psi_t(\sum_{i=1}^{N}\theta_{it}{}^{j}\,\overline{W_{it}}) \leq 0,$$

$$\forall j, \forall i, \forall t$$

$$X_{it} \geq 0, \quad I_{it} \geq 0, W_{it} \geq 0, R_{it} \geq 0, \quad \forall i, \forall t,$$

where $j = 1, \dots, M$, stand for M different resources used in the production process.

Let's take a closer look at this model, mainly focusing on the set of constraints

$$\xi_{it}{}^{j}X_{it} - \rho_{it}{}^{j}\Psi_t(\textstyle\sum_{i=1}^{N}\theta_{it}{}^{j}\,\overline{W_{it}}) \leq 0, \quad \forall j, \forall i, \forall t \tag{35}$$

since the others are classical linear balance flow constraints. For the sake of simplicity, suppose that

$\xi_{it}{}^{j}X_{it} = \theta_{it}{}^{j} = \alpha, \forall i, \forall t, \forall j$, and then,

$$\alpha X_{it} - \rho_{it}{}^{j}\Psi_r(\textstyle\sum_{i=1}^{N}\alpha \hat{W}_{it}) \le 0, \quad \forall j, \forall i, \forall t. \tag{36}$$

In the cases where $\Psi_r(.)$ is a homogeneous function these inequalities simplify to,

$$X_{it} - \rho_{it}{}^{j}\Psi_r(\textstyle\sum_{i=1}^{N}\hat{W}_{it}) \le 0, \quad \forall j, \forall i, \forall t. \tag{37}$$

which make it clear that the quantity of products i produced at period t, X_{it}, depends not only on the WIP related directly to product i, but also to the overall workload state of the system, which is quite a revealing insight about the performance of a system with workload resources. This conclusion suggests, in the end, that capacity is less a matter of a throughput issue than a state of the productive system, therefore, unveiling that, in the end pricing capacity is related to the overall performance of the productive system and not just about the quantity of products it can manufacture. It is a property of the productive system not of the production of the system.

Model (34) is a convex model, since all the functions are convex, and the constraints of equality are linear. So, the existence of an interior feasible solution assures that the Slater condition holds, and then, strong duality, and everything from the previous analysis in Section 3.4., applies.

3. NUMERICAL ILLUSTRATION AND FINAL REMARKS

Estimating if or when to increase or decrease production capacity has been one of the secret keys of the production schedule, both from the point of view of the efficiency of the production system as well as from the point of view of the economy of the productive system itself. Linear programming models are generally not very helpful in relation to this issue in that their dual variables, which give information about prices, are always kept at zero while the productive system does not completely use its total capacity.

From queuing theory and practical experience from the shop floor, we know that systems in general start degrading early before a 100% of utilization; therefore, linear programming models must be modified to deal with the issue of pricing for lower levels of capacity utilization. To capture the economic aspect, not always predicted by linear programming, we must introduce certain modifications to the linear programming models, which frequently destroy their linearity, as for instance, in the case of introducing the CF function to price low levels of capacity utilization. To go from the modified model to a new LP model there is almost always a price to be paid, which frequently is to face a very large linear programming model as can be seen in Ali *et al.* [1] or Missbouer and Uzsoy [7].

However, the convex model provided by the CF function modification of the linear programming model is very simple and can be treated directly, since convex duality under Slater's condition is an exact duality, which will allow for treating the Karush-Kuhn-Tucker penalty parameters as estimated prices. The convex model also shares with linear programming models the aspect that local solutions are global solutions, and the set of optimal solutions are convex sets.

The numerical illustration exposed below considers only the convex CF model modification from the LP model, and we suggest to those who want to see the modified large linear programming models to consult [1] or [7] for further information.

3.1 Numerical Illustration

To illustrate the role of the CF function in pricing low levels of capacity utilization we present a small example to emphasize some evidence predicted by the convex model, and revealed by numerical experiments, such as the changeovers of penalty parameters upon different distributions of capacity in the planning horizon. The complete experiment has nineteen Tables, presented in Appendix A. The few tables presented in this Section show the data for the experiment and the most serious of the consequences of the lack of harmony between production planning and capacity planning: infeasibility.

The Table of Data bellow shows that the global nominal capacity in the planning horizon is 20% above the estimated demands, and different ways in the distribution of capacity in the planning horizon to evidence the role of correlation between capacity and demand in the same period, and not only in the whole horizon. The consequence of these distributions clearly suggest that planning production schemes requires a fine line with planning of capacity, in addition to a planning for releases. Although this fact was knew, there not exist a general explicitly way to measured it.

Furthermore, optimal solutions requires a sensitiveness evaluation to avoid misinterpretation of the CF function.

Table 1 presents the used data and all the cases considered for distribution of capacity. The existence of nonzero dual variable related to capacitysuggests a mismatch in the capacity allocated for the period. For each of the cases we perturbed the CF function in 10% to estimate sensitiveness. The numerical results of these perturbations are show for each perturbed case: 90% of nominal capacity and 110% of nominal capacity. In some cases, response for perturbation is dramatic.

Table 1. Table of Data

Cost	Period					
Parameters	1	2	3	4	5	6
Production Cost	10	10	10	10	10	10
Inventory Cost	0.6	0.6	0.7	0.7	0.8	0.8
WIP Cost	3	3	3	3	3	3
Release Cost	2	2	2	2	2	2
Demands	100	250	300	280	200	290
Capacity 1st Case	120	300	360	336	240	348
Capacity 2nd Case	300	360	336	240	348	120
Capacity 3rd Case	360	336	240	348	120	300
Capacity 4th Case	336	240	348	120	300	360
Capacity 5th Case	240	348	120	300	360	336
Capacity 6th Case	348	120	300	360	336	240

Table 2. - Capacity 1st Case, presents the unperturbed Capacity, and since there are enough capacity in all the periods, the dual variables are zero, as expected.

Table 2. 1st Case

Optimal Solution Value: 17040.00	Period					
	1	2	3	4	5	6
Production	100	250	300	280	200	290
Inventory	0	0	0	0	0	0
WIP	0	0	0	0	0	0
Release	100	250	300	280	200	290
Demands	100	250	300	280	200	290
Nominal Capacity	120	300	360	336	240	348
Dual Variables	0	0	0	0	0	0

Table 3. - Perturbed 1st Case, presents the perturbation in that capacity is held in 90% of nominal Capacity. The optimal solution requires that release in the first period is bigger than in the unperturbed case, and then WIP is positive and so, the first dual variable, which suggest lack of capacity in the next period.

Table 3. Perturbed 1st Case

Optimal Solution Value: 17115.00	Period					
	1	2	3	4	5	6
Production	100	250	300	280	200	290
Inventory	0	0	0	0	0	0
WIP	25	0	0	0	0	0
Release	125	225	300	280	200	290
Demands	100	250	300	280	200	290
90% of Capacity	108	270	324	302	216	313
Dual Variables	50.6250	0	0	0	0	0

When the perturbation of capacity is positive, 110% of nominal Capacity all the dual variables are zero, as expected. See Appendix A.

Table 4. 5th Case

Optimal Solution Value: 17098.40	Period					
	1	2	3	4	5	6
Production	202	338	110	280	200	290
Inventory	102	190	0	0	0	0
WIP	0	0	0	0	0	0
Release	202	338	110	280	200	290
Demands	100	250	300	280	200	290
Nominal Capacity	240	348	120	300	360	336
Dual Variables	0	0.2059	0.4363	0	0	0

Table 5. 5th Case

Optimal Solution Value: ?	Period					
	1	2	3	4	5	6
Production	216	313	108+179=287			
Inventory	116	179				
WIP	0	0	Infeasibility!			
Release	216	313				
Demands	100	250	300			
90% Capacity	216	313	108			
Dual Variables	-	-	-	-	-	-

Table 4 – Perturbed 5th Case, shows a rather balanced capacity related to required demand, except for the 3rd period, and its overall production cost compares with the best first case of Table 2. However, it responds dramatically to perturbation (shortage) of capacity with infeasibility, as shown in the 3rd column of Table 5. Production planning requires that production should be greater than installed capacity in that period to satisfy demand, thus producing infeasibility.

The incorporation of clearing function in the model for planning production actually works as a perturbation about available capacity imposing a reduction on nominal capacity, i.e., clearing function take in to account variability, thus reducing available capacity.

3.2 Final Remarks

The use of clearing function in models of linear programming actually modify the amount of available capacity to match demand. The way the new convex model responds to changes about capacity seems to suggest that not only lead times varies nonlinearly when resources are scarce, but also the penalty parameters (dual variables) and hence, production costs. Therefore, optimal production planning is not divorced from a plan of allocation of capacity and a plan of releases of productions orders as well. The response of the model is obvious: unlike the common sense, the optimal functioning of the productive systems occur well below their levels of nominal capacity, if the concept of clearing function is incorporate into the model. And this incorporation is supposed to improve the model.

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APPENDIX A

Table 2.1st Case

Optimal Solution Value: 17040.00	Period					
	1	2	3	4	5	6
Production	100	250	300	280	200	290
Inventory	0	0	0	0	0	0
WIP	0	0	0	0	0	0
Release	100	250	300	280	200	290
Demands	100	250	300	280	200	290
Nominal Capacity	120	300	360	336	240	348
Dual Variables	0	0	0	0	0	0

Table 3. Perturbed 1st Case

Optimal Solution Value: 17115.00	Period					
	1	2	3	4	5	6
Production	100	250	300	280	200	290
Inventory	0	0	0	0	0	0
WIP	25	0	0	0	0	0
Release	125	225	300	280	200	290
Demands	100	250	300	280	200	290
90% of Capacity	108	270	324	302	216	313
Dual Variables	50.6250	0	0	0	0	0

Table 4. Perturbed 1st Case

Optimal Solution Value: 17040.00	Period					
	1	2	3	4	5	6
Production	100	250	300	280	200	290
Inventory	0	0	0	0	0	0
WIP	0	0	0	0	0	0
Release	100	250	300	280	200	290
Demands	100	250	300	280	200	290
110% Capacity	132	330	396	369	264	382
Dual Variables	0	0	0	0	0	0

Table 5. Perturbed 2nd Case

Optimal Solution Value: 17147.40	Period					
	1	2	3	4	5	6
Production	100	316	326	230	338	110
Inventory	0	66	92	42	180	0
WIP	0	0	0	0	0	0
Release	100	316	326	230	338	110
Demands	100	250	300	280	200	290
Nominal Capacity	300	360	336	240	348	120
Dual Variables	0	0	0.2061	0.5217	0.8236	1.2000

Table 6. Perturbed 1st Case

Optimal Solution Value: 17228.80	Period					
	1	2	3	4	5	6
Production	207	314	292	206	303	98
Inventory	107	171	163	89	192	0
WIP	0	0	0	0	0	0
Release	207	314	292	206	303	98
Demands	100	250	300	280	200	290
90% Capacity	270	324	302	216	313	108
Dual Variables	0	0.2063	0.4136	0.7339	1.0330	1.4326

Table 7. Perturbed 2nd Case

Optimal Solution	Period					
Value: 17121.00	1	2	3	4	5	6
Production	182	254	372	122	200	290
Inventory	82	86	158	0	168	0
WIP	0	0	0	0	0	0
Release	182	254	372	122	200	290
Demands	100	250	300	280	200	290
110% Capacity	330	396	369	264	382	132
Dual Variables	0	0.2078	0.4107	0.7573	0	0

Table 8. 3th Case

Optimal Solution	Period					
Value: 17102.20	1	2	3	4	5	6
Production	126	326	230	338	110	290
Inventory	26	102	32	90	0	0
WIP	0	0	0	0	0	0
Release	126	326	230	338	110	290
Demands	100	250	300	280	200	290
Nominal Capacity	360	336	240	348	120	300
Dual Variables	0	0.2061	0.4173	0.7207	1.0900	1.3448

Table 9. Perturbed 3th Case

Optimal Solution	Period					
Value: 17193.88	1	2	3	4	5	6
Production	260.4	292.4	206	303.2	98	260
Inventory	160.4	202.8	108.8	132	30	0
WIP	0	0	0	0	0	0
Release	260.4	292.4	206	303.2	98	260
Demands	100	250	300	280	200	290
90% Capacity	324	302	216	313	108	270
Dual Variables	0	0.2068	0.4194	0.7230	1.1020	1.3500

Table 10. Perturbed 3th Case

Optimal Solution	Period					
Value: 17072.60	1	2	3	4	5	6
Production	100	296	254	358	122	290
Inventory	0	46	0	78	0	0
WIP	0	0	0	0	0	0
Release	100	296	254	358	122	290
Demands	100	250	300	280	200	290
110% Capacity	396	369	264	382	132	330
Dual Variables	0	0	0.2078	0	0.3245	0

Table 11. 4th Case

Optimal Solution	Period					
Value: 17147.80	1	2	3	4	5	6
Production	252	230	338	110	200	290
Inventory	152	132	170	0	0	0
WIP	0	0	0	0	0	0
Release	252	230	338	110	200	290
Demands	100	240	348	120	300	360
Nominal Capacity	336	240	348	120	300	360
Dual Variables	0	0.2086	0.4118	0.7636	0	0

Table 12. Perturbed 4th Case

Optimal Solution	Period					
Value: 26538.03	1	2	3	4	5	6
Production	300.0157	213.9850	309.7808	106.2185	264.6000	225.4000
Inventory	81.20	85.20	158	0	0	0
WIP	0	0	0	0	0	0
Release	1258.305	103.6913	58.0040	0	0	0
Demands	100	250	300	280	200	290
90% Capacity	396	369	264	382	132	330
Dual Variables	1595.830	1596.030	1599.200	1601.205	2.700	0

Table 13. Perturbed 4th Case

Optimal Solution	Period					
Value: 17120.68	1	2	3	4	5	6
Production	181.20	254	372.80	122	200	290
Inventory	81.20	85.20	158	0	0	0
WIP	0	0	0	0	0	0
Release	181.20	254	372.80	122	200	290
Demands	100	250	300	280	200	290
110% Capacity	369	264	382	132	330	396
Dual Variables	0	0.2078	0.4107	0.7573	0	0

Table 14.th Case

Optimal Solution	Period					
Value: 17098.40	1	2	3	4	5	6
Production	202	338	110	280	200	290
Inventory	102	190	0	0	0	0
WIP	0	0	0	0	0	0
Release	202	338	110	280	200	290
Demands	100	250	300	280	200	290
Nominal Capacity	240	348	120	300	360	336
Dual Variables	0	0.2059	0.4363	0	0	0

Table 15. Perturbed 5th Case

Optimal Solution	Period					
Value: 31012.32	1	2	3	4	5	6
Production	214.4895	310.6233	(138.4098)	266.4774	317.52	172.48
Inventory	114.4895	175.1128	13.5226	0	117.52	0
WIP	1205.51	894.8872	756.4774	490	172.48	0
Release	1420					
Demands	100	250	300	280	200	290
90% Capacity	216	313	108	270	324	302
Dual Variables	-	-	-	-	-	-

Table 16. Perturbed 5th Case

Optimal Solution	Period					
Value: 17086.64	1	2	3	4	5	6
Production	155.2	372.8	122	280	200	290
Inventory	55.2	178	0	0	0	0
WIP	0	0	0	0	0	0
Release	155.2	372.8	122	280	200	290
Demands	100	250	300	280	200	290
110% Capacity	264	382	132	330	396	369
Dual Variables	0	0.2053	0.4327	0	0	0

Table 17. 6th Case

Optimal Solution	Period					
Value: 17090.00	1	2	3	4	5	6
Production	250	110	290	280	260	230
Inventory	150	10	0	0	60	0
WIP	0	0	0	0	0	0
Release	250	110	290	280	260	230
Demands	100	250	300	280	200	290
Nominal Capacity	348	120	300	360	336	240
Dual Variables	0	0.2181	0.4137	0	0	0.3130

Table 18. Perturbed 6th Case

Optimal Solution	Period					
Value: 17111.60	1	2	3	4	5	6
Production	292	98	260	280	284	206
Inventory	192	40	0	0	84	0
WIP	0	0	0	0	0	0
Release	292	98	260	280	284	206
Demands	100	250	300	280	200	290
90% of Capacity	313	108	270	324	302	216
Dual Variables	0	0.2204	0.4153	0	0	0.3145

Table 19. Perturbed 6th Case

Optimal Solution Value: 17076.40	Period					
	1	2	3	4	5	6
Production	228	122	300	280	236	254
Inventory	128	0	0	0	36	0
WIP	0	0	0	0	0	0
Release	228	122	300	280	236	254
Demands	100	250	300	280	200	290
110% of Capacity	382	132	330	396	396	264
Dual Variables	0	0.2163	0	0	0	0.3118

Upotreba funkcije obračuna za procenu kapaciteta naplate resorsa na lošim nivoima upotrebe u proizvodnom planiranju

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Rezime

Veoma je poznato iz svakodnevnog industrijskog iskustva da visoki nivoi propusnih kapaciteta u proizvodnji zavise od visokih nivoa radova u toku ili ubacivanja u sistem, i da visoki nivoi radova u toku mogu da povećaju celokupno vreme od početka do kraja procesa u sistemu, smanjujući očekivani obrt. Ovo jasno ukazuje da ponekad povećanje kapaciteta proizvodnje jeste u našem najboljem interesu čak i pre nego što postane vremenski prekratko, iako neophodne informacije da bi se ovo postiglo nisu obezbeđene u većini pristupa koji se koriste za ovu temu, i posebno, u klasičnim linearnim programskim modelima. Nedavno su neki autori razvili okvir kako bi zaobišli ovaj nedostatak u pristupu linearnog programiranja, zasnovan na konceptu obračunske funkcije koja teži, zajedno sa pristupom linearnog programiranja, da dozvoli naplatu loših nivoa upotrebe kapaciteta. Ipak, nov model nije tretiran direktno već je nastao kao aproksimacija linearnog modela, koji je imao klasičan tretman i otkrivao jako malo informacija. U ovom radu mi tretiramo nov model direktno, i još više, imamo nov pristup za linearni model, koji je po našem mišljenju stvorio novu i dublju verziju ove teme.

Ključne reči: *funkcija obračuna, konveksno programiranje, upotreba kapaciteta, rad u toku*