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A new hybrid algorithm for solving the vehicle routing problem with route balancing

S.H. Huang[a], Y.H. Huang[b,*], H.C. Lee[a], Y.Y. Tong[a]

[a] *National Kaohsiung University of Science and Technology, Department of Logistics Management, Kaohsiung, Taiwan;*
[b] *National Yunlin University of Science and Technology, Department of Civil and Construction Engineering, Yunlin, Taiwan*

ABSTRACT

This paper addresses a vehicle routing problem with route balancing to minimize the total travel cost and equity measurement. We propose a hybrid method combining Particle Swarm Optimization and Ant Colony Optimization with the global search characteristic of PSO and the path-finding ability of ACO. The proposed method first solves the benchmark instances to obtain the total travel distance and the equity measurement value. Then, by considering predefined threshold values of the equity measurement in the original solution, the vehicle routing problem with route balancing can be solved using the proposed method. Experiments are conducted to obtain better-balanced routes by considering more than one equity measurement. The results showed that this hybrid mechanism is promising to become a better method of VRP.

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*Corresponding author:
Ying-Hua Huang
huangyh@yuntech.edu.tw

1. Introduction

Vehicle routing problem (VRP) is one of the essential issues in logistics management. A traditional VRP is to dispatch vehicles from the depot to serve all the customers and then return to the depot. Various vehicle routing problems have been developed to cope with different situations, such as vehicle routing problems with time window constraints [1], heterogeneous fleet vehicle routing problems [2], the heterogeneous fixed fleet in an open vehicle routing problem [3], vehicle routing problem with multi-depots [4, 5, 6]. Other excellent research related to different topics, including multi-compartment [7, 8], pickup and delivery [9, 10, 11], feeder VRP [12], and electric VRP [13], can also be found in the literature. In addition to the conventional deterministic routing problems, some concentrated on solving the VRP with stochastic demand, such as vehicle routing problems with stochastic demand and duration constraints [14] and inventory routing problems with demand uncertainty [15]. Other outstanding research, such as Laporte et al. [16] and Rei et al. [17], can also be found in the literature.

All the versions of VRP mentioned above aim to minimize the total travel cost. However, from a mana-

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gerial perspective, dispatching vehicles with balanced routes is more appropriate. More balanced routes indicate that all routes are more similar regarding equity metrics such as route length. The route balancing problem tries to even out the workload distribution. It is essential for workforce planning that all the routes have near working time so that drivers' shifts are close to each other.

Solving the vehicle routing problem with route balancing (VRPRB) is to yield more balanced routes; however, the total travel length will inevitably be lengthened [18]. If the route balance is the only thing concerned, the entire travel length could be prolonged tremendously. The output results of Borgulya [19] demonstrated this phenomenon. It is a trade-off between the total route length and the route balance. This study proposes a combinatorial algorithm of Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO) to solve the VRPRB by considering various equity measurements to examine the proposed algorithm's feasibility. Meanwhile, experiments are conducted to figure out the balance point between the total travel distance and the route balance.

The remaining sections are organized as follows. Section 2 briefly reviews research related to VRPRB. In section 3, the model for VRPRB is presented. Section 4 consists of the introduction of PSO and ACO, as well as the proposed solving process. Section 5 provides the experiments, the computational results, and the sensitivity analysis of the solving process. Finally, section 6 concludes the research findings and suggests future research.

2. Related works of vehicle routing problem with route balancing

Vehicle routing problem with route balancing (VRPRB) is generally modeled as a bi-objective or multi-objective optimization problem [20, 21] in which the main objective is the total travel cost, and the other is the equity. Equity measurement indicates the equalization level of routes, which also reflects workload balance. For instance, Oyola and Lokketangen [22] adopted the difference between the maximum and minimum route distances (the range) as the second objective to solve the route balancing problem, as well as Jozefowiez et al. [23]. Keskinturk and Yildirim [24] tackled the bakery distribution VRP by minimizing the average sum of relative imbalance. They used the term "work balance" to indicate that the model balances the route working time, which comprises the travel and loading time. Lozano et al. [25] discussed equity as the cumulative difference (of all routes) between the given route and the minimum route and variance of route lengths. Jingjing et al. [26] considered equity as the maximum and minimum workload ratio. In their research, the workload combines route length and load. In Linfati et al.'s study [27], the workload balancing is based on the deviation concerning the average load of the routes.

Another way to consider the balanced routes is to deal with a min-mix vehicle routing problem (min-max VRP). A min-max VRP is to find a solution in which the cost of the longest route is minimized instead of finding the solution with the least total travel cost. Minimizing the maximal route length implies balancing the routes. This objective could be more suitable for some circumstances, such as disaster relief efforts, computer networks, and workload balance. To the best of the authors' knowledge, the number of related research using min-max as equity measurement is more than other indicators. For example, Golden et al. [28] were probably one of the earliest attempts to solve the min-max VRP. They proposed Tabu search and adaptive memory heuristics as the solving process to generate good quality solutions within reasonable computer processing time. Bertazzi et al. [29] analyzed the conventional VRP with a minimum total length objective (min-sum) and the min-max VRP in the worst case.

Some researchers studied the variants of min-max VRP for different circumstances. For instance, Yakici [30], based on the Ant Colony Optimization method, proposed a heuristic approach to solve the min-max VRP with mixed fleet and demand. Rabbani et al. [31] solved a bi-objective and multi-depot VRP with time windows by mixed integer non-linear programming. Narasimha et al. [32] and Wang et al. [33] worked on the min-max multi-depot VRP and solved it using the Ant Colony Optimization technique and a heuristic approach, respectively. Wang et al. [34] developed a multi-period workload balancing problem under stochastic demand and dynamic daily dispatching.

This paper proposes a combinatorial algorithm of Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO) to solve the VRPRB. We also solve the VRPRB by combining the total travel distance and the equity measurements as the objective function so that the travel distance and equity measurements can be optimized simultaneously.

3. Problem description

Let $G=(V,E)$ represents a graph in a plane where $V=\{v_0,\ldots,v_n\}$ denotes the set of vertex and $E=\{(v_i,v_j):v_i,v_j\in V\}$ denotes the set of edges. Vertex v_0 represents the depot, and the remaining vertices correspond to customers. Each customer v_i is associated with a non-negative demand d_i. Also, each edge (v_i,v_j) is associated with a non-negative cost c_{ij}. A set of vehicles K is located at the depot initially, and each vehicle has an identical capacity, Q. The VRP determines a set of vehicle routes that start and end at the depot and visit customers during its journey without violating any constraints such as flow conservation and capacity constraints. The objectives of the VRPRB are to minimize the range (ζ) and optimize the total travel cost.

Sets:

V Set of vertex

K Set of vehicles

S Subset of V

Model parameters:

c_{ij} Travel cost of arc (i, j)

r^{max} Route with max cost among the routes.

$$r^{\max}=\max\Big(\sum_{i\in V}\sum_{j\in V,i\neq j}c_{ij}x_{ij}^{k}\quad \forall k\in K\Big)$$

r^{min} Route with min cost among the routes.

$$r^{\min}=\min\Big(\sum_{i\in V}\sum_{j\in V,i\neq j}c_{ij}x_{ij}^{k}\quad \forall k\in K\Big)$$

d_j The demand of node j

Q Vehicle capacity

ζ The range; the distance difference between r^{max} and r^{min}

z Total travel cost

ε Predefined threshold value

Indices:

i, j Indexes for nodes

k Index for vehicle

Decision variable:

x_{ij}^k Binary variable, = 1 if the vehicle k travels from node i to node j, 0 otherwise

Model formulation

$$\min\quad z=\sum_{i\in V}\sum_{j\in V,i\neq j}\sum_{k\in K}c_{ij}x_{ij}^{k} \tag{1}$$

$$\min\quad \zeta=r^{\max}-r^{\min} \tag{2}$$

s.t.

$$\sum_{i\in V}\sum_{k\in K}x_{ij}^{k}=1\quad \forall j\in V,i\neq 0,j\neq 0,i\neq j \tag{3}$$

$$\sum_{j\in V}\sum_{k\in K}x_{ij}^{k}=1\quad \forall i\in V,i\neq 0,j\neq 0,i\neq j \tag{4}$$

$$\sum_{i\in V}\sum_{k\in K}x_{i0}^{k}=\sum_{j\in V}\sum_{k\in K}x_{0j}^{k}=|K|\quad \forall i,j\in V,i\neq 0,j\neq 0 \tag{5}$$

$$\sum_{i\in V}\sum_{j\in V}d_j x_{ij}^{k}\le Q\quad \forall k\in K,i\neq j \tag{6}$$

$$\sum_{i\in S}\sum_{j\in S}x_{ij}^{k}\le |S|-1\quad \forall S\subseteq V\setminus v_0,\forall k\in K \tag{7}$$

$$r^{\max}\ge\sum_{i\in V}\sum_{j\in V,i\neq j}c_{ij}x_{ij}^{k}\quad \forall k\in K \tag{8}$$

$$r^{\min}\le\sum_{i\in V}\sum_{j\in V,i\neq j}c_{ij}x_{ij}^{k}\quad \forall k\in K \tag{9}$$

$$x_{ij}^{k}\in\{0,1\}\quad \forall i,j\in V,\forall k\in K \tag{10}$$

The objective functions of the problem are formulated as Equations (1) and (2). Equation (1) is the total travel distance as conventional VRP, and Equation (2) is the difference between the maximum and minimum route distances (the range). Constraints (3) and (4) ensure that each customer is visited once and only once. Constraint (5) indicates that each vehicle departs from the depot and returns to the depot. Constraint (6) is the vehicle capacity constraint; the total demand on the route shall not exceed the vehicle capacity. Constraint (7) is the sub-tour elimination constraint. Constraint (8) circumscribes the route with the longest route distance, and Constraint (9) defines the route with the shortest route length among all the routes. These two parameters (r^{max} and r^{min}) are auxiliaries for ζ. The parameter ζ is a general term for measuring route balance, but not limited to this indicator. Constraint (10) denotes that x_{ij}^k is a decision variable.

Generally, the first objective z will be lengthened while the second objective ζ gets shortened. That means these two objectives conflict with each other. Such a phenomenon makes mathematics programming challenging to be solved. Thus, we modify the mathematics program by removing the second objective (i.e., Equation (2)) and revise it as a constraint.

$$\zeta\le\varepsilon \tag{11}$$

Equation (11) indicates that the equity measurement (the range) is sufficiently acceptable as long as this constraint is satisfied, meaning that the range is less than a predefined threshold value ε. This revised model can avail us to solve the problem.

4. Methodology

This study proposes a hybrid algorithm combining PSO and ACO to solve the VRPRB. This proposed hybrid mechanism combines the global search characteristic of PSO and the path-finding ability of ACO.

PSO was initially proposed by Kennedy and Eberhart [35], inspired by the movement of organisms such as bird flocks or fish schools. Each particle has a speed moving toward the current best solution. The velocity and the position of each particle are changed according to the following equations.

$$v_i^{k+1} = \omega v_i^k + \varphi_1 \times r_1 \times (P_{best} - x_i^k) + \varphi_2 \times r_2 \times (G_{best} - x_i^k) \tag{12}$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \tag{13}$$

Parameter x_i^k denotes the position of a particle and v_i^k indicates the speed of the particle on the kth iteration. Also, P_{best} and G_{best} represent the best solutions found by a particle and globally, respectively. The parameter ω is the inertia weight to manage the trade-off between the swarm's global and local exploration abilities. Besides, parameters ϕ_1 and ϕ_2 are acceleration factors to control the relative attraction of P_{best} and G_{best}, respectively. Moreover, r_1 and r_2 are random numbers from 0 to 1. The particles will move in the search space based on Equations (12) and (13) toward the best solution. Better solutions are anticipated to be found during the movements.

The position of a particle in the solution domain indicates one feasible solution. Every movement of one particle infers that the particle moves from one solution to an adjacent solution in the search space. In this study, the ACO is recruited to manipulate particle movement. The ACO used in this study is based on the Ant Colony System (ACS) proposed by Dorigo and Gambardella [36], constituted by three main aspects, i.e., state transition rule, local pheromone updating rule, and global pheromone updating rule.

- State transition rule

The state transition rule is used to determine the next stop for an ant and is formulated as Equation (14).

$$s = \begin{cases} \max\limits_{j \in \Omega_i} \{Att_{ij}\} & if \quad q \le q_0 \\ p_{ij} & otherwise \end{cases} \tag{14}$$

Set Ω_i contains all feasible nodes for ant i and Att_{ij} indicates the attraction of arc (i, j). The attraction of arc (i, j) can be calculated by Equation (15).

$$Att_{i,j} = (\tau_{ij})^{\alpha} (\eta_{ij})^{\beta} \tag{15}$$

Parameters τ_{ij} and η_{ij} denote the amount of pheromone and the reciprocal of the distance between nodes i and j, respectively. The corresponding exponents α and β control the influence of the pheromone value allocated on arc (i, j) and the desirability of arc (i, j), respectively. The parameter q is a random number, and $0<q_0<1$ is predetermined. The next traveling arc will be the one with maximum attraction while $q<q_0$ holds. Otherwise, the next traveling arc of an ant will be determined by the state transition probability p_{ij}, which can be calculated by Equation (16).

$$p_{ij} = \begin{cases} \dfrac{Att_{ij}}{\sum\limits_{k \in \Omega_i} Att_{ik}} & \forall j \in \Omega_i \\ 0 & \forall j \notin \Omega_i \end{cases} \tag{16}$$

- Local pheromone update

The ants deposit a tiny amount of pheromone on the path they traversed, and the pheromone is evaporated with time. It is called the local pheromone update. Assume $0<\rho<1$ denotes the evaporation rate and $\tau_0=1/(N \cdot J_{\psi^{ini}})$ is the initial trail intensity in which N is the total number of nodes and $J_{\psi^{ini}}$ is the cost of the initial solution. The local pheromone update rule is represented as Equation (17).

$$\tau_{i,j} = (1-\rho) \cdot \tau_{i,j} + \rho \cdot \tau_0 \tag{17}$$

- Global pheromone update

After a preordained number of iterations, all arcs' pheromones are updated according to the current optimal route ψ^{gl} , and it is called the global pheromone update. The pheromone levels on the arcs are reset to their initial status, and the arcs constituting ψ^{gl} will be deposited with a specific amount of pheromone, according to Equation (18). The parameter $J_{\psi^{gl}}$ is the cost of the best solution ψ^{gl} at present. This rule resets the ant colony's situation to an optimal start so that it has a higher probability of approaching a better solution.

$$\tau_{ij} = (1-\rho) \cdot \tau_{ij} + \rho / J_{\psi^{gl}} \quad (i, j) \in \psi^{gl} \tag{18}$$

These two methods, PSO and ACO, are combined as a hybrid algorithm named Ant Swarm Optimization (ASO). The structure of ASO is mainly based on PSO. A particle contains the best solution of the particle and a matrix for the pheromone deposited. The ACO is recruited to change the position of a particle. Each particle contains one ant that will conduct the solution searching. In this study, the pheromone deployment of one solution is treated as the "eigenvalue" of particle position in the search space. Pheromones are superimposed on the trails of the global best solution (G_{best}), the particle best solution (P_{best}), and the particle current solution for each particle. Parameters v_1, v_2, and v_3 are the weights of G_{best}, P_{best}, and the current solution (ψ), respectively, to control the moving speed and direction of the particle. That means the amount of pheromone deposited on an arc in the process of the local pheromone update scheme will be as Equations (19) to (21).

$$\tau_{i,j} = ((1-\rho)\cdot\tau_{i,j} + \rho\cdot\tau_0)\times v_1 \quad \forall(i,j)\in\psi^{G_{best}} \tag{19}$$

$$\tau_{i,j} = ((1-\rho)\cdot\tau_{i,j} + \rho\cdot\tau_0)\times v_2 \quad \forall(i,j)\in\psi^{P_{best}} \tag{20}$$

$$\tau_{i,j} = ((1-\rho)\cdot\tau_{i,j} + \rho\cdot\tau_0)\times v_3 \quad \forall(i,j)\in\psi \tag{21}$$

Thus, a new solution generated through these pheromone distributions can be treated as a particle moving from its current position to the best solution. Figure 1 depicts the flowchart of ASO. After setting up the graph and all the constants, such as P (number of particles), and R (number of rounds), an ant of one particle is dispatched to create the initial solution. A round means that all the ants in all the particles made

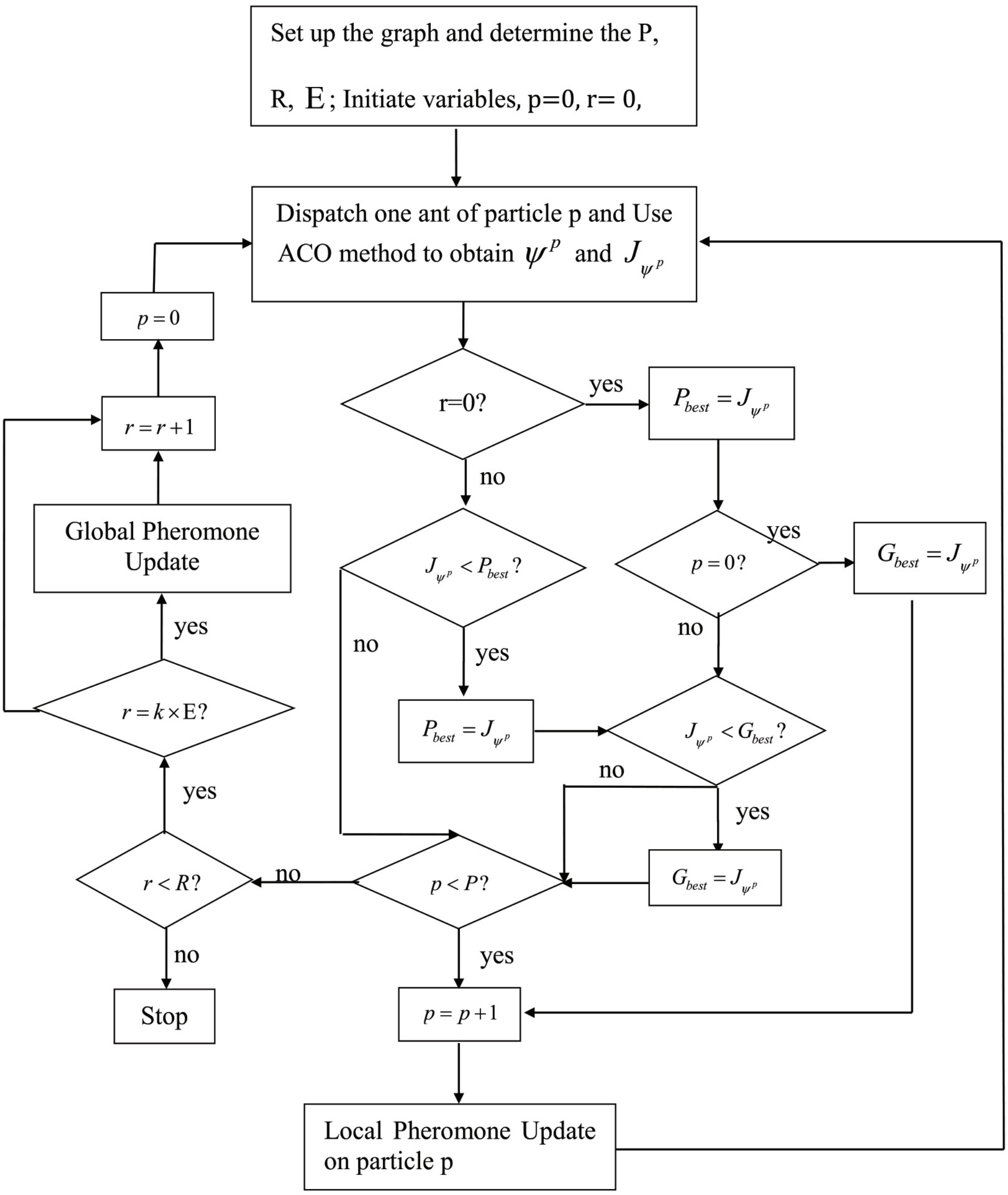

Figure 1. Flowchart of ASO

one move and created one new solution. If an ant can yield a better solution than the corresponding particle's best solution (P_{best}) or the global best solution (G_{best}), the new solution will substitute for the P_{best} or G_{best}. The local pheromone update scheme is conducted right after a move, and the global pheromone update scheme is conducted when the number of rounds is a multiple of a predefined constant (E). This solving process stops as soon as the number of rounds equals a predefined number R, and the G_{best} recorded is the final solution obtained.

5. Experiment design and discussion

5.1: verifying ASO by solving capacitated vehicle routing problem

The test problems choose Instances 1-5 and 11, 12 of Christofides et al. [37] to verify the quality of the solution process. These Capacitated Vehicle Routing Problem (CVPR) problems contain customers ranging from 50 to 199. The parameter settings related to ACO are as follows. Parameter q_0 is set to 0.3, ρ is set to be 0.2, as well as $\alpha \in [2, 3, 4, 5]$ and $\beta \in [2, 3, 4]$ according to the experiments of parameter examining. The global pheromone update is conducted every 30 rounds (E=30). In addition to the parameter settings related to ACO, the parameters related to PSO are as follows. The number of particles is set to 30, and the speed control parameters v_1, v_2, and v_3 are set to 1/3. The solving process stops as soon as the total moving rounds equals 900. A program based on the proposed algorithm is coded using the computer language Python and executed on a desktop with Inter(R) Core(TM) i7-4790 CPU@ 3.60GHz and 8GB RAM. We then apply this program to solve the benchmark dataset to examine the performance of the solving process. The outputs are listed in Table 1.

The solutions of the CMT1 and CMT12 are equal to the optimum solutions, as shown in Table 1. Also, the difference ratio (dr) of other instances' solutions and optimum solutions are less than 1%, except for CMT4 and CMT5. It is observed that problem size is a critical factor in affecting performance. These output results indicate that the proposed method is a promising algorithm for solving the problems.

5.2: using ASO to solve VRPRB

In order to minimize the travel cost and the equity measurement (the range between maximum and minimum route distances) simultaneously, the objective function is modified as Equation (22) while solving the VRPRB where ν is the weight of the equity measurement. Equation (22), the objective function, is constituted by the total travel distance and weighted equity measurement. This objective can minimize the total travel distance and equity measurement simultaneously.

$$\min \sum_{i \in V} \sum_{j \in V, i \neq j} \sum_{k \in K} c_{ij} x_{ij}^k + \nu \zeta \tag{22}$$

To examine the performance of the ASO in solving the VRPRB, we set the threshold values of equity measurement (ε) shown in the constraint Equation (11). The values of 3/4, 1/2, and 1/4 of the range between the maximum and the minimum route distances found in the CVRP, which is the $\overline{\overline{r^{\max} - r^{\min}}}$ value listed in Table 1, are used as the threshold values to solve the route balancing problems accordingly. The output results are listed in Table 2.

It can be observed from Table 2 that the total travel distance (TD) is increased along with the decrement of the difference between the maximum and the minimum route distances ($\overline{r^{\max} - r^{\min}}$) for VRPRB, as we expected. When considering the value of ε equals 3/4 of $\overline{\overline{r^{\max} - r^{\min}}}$, the solution can be easily

Table 1. The results of the CVRP problems using ASO

Instance	n	Q	v	α	β	os	s	r^{max}	r^{min}	$\overline{\overline{r^{\max} - r^{\min}}}$	dr
CMT1	50	160	5	3	3	524.61	524.61	118.52	98.45	20.07	0%
CMT2	75	140	10	3	2	835.26	840.94	120.16	42.67	77.49	0.68%
CMT3	100	200	8	2	2	826.14	827.39	126.90	59.35	67.55	0.15%
CMT4	150	200	12	4	3	1028.42	1055.00	129.17	39.08	90.09	2.58%
CMT5	199	200	17	4	4	1291.44	1401.94	137.99	20.80	117.19	8.56%
CMT11	120	200	7	5	2	1042.11	1043.65	213.63	66.96	146.67	0.15%
CMT12	100	200	10	3	3	819.56	819.56	137.02	43.59	93.43	0%

Note: n: number of customers; Q: vehicle capacity; v: number of vehicles; os: optimum solution; s: solution achieved by ASO; r^{max}: the max cost among the routes; r^{min}: the min cost among the routes; $\overline{\overline{r^{\max} - r^{\min}}}$ = r^{max} - r^{min}; dr: difference ratio, $\frac{s-os}{os} \times 100\%$.

obtained with a related smaller equity measurement weight ν along with less than 5% of the travel distance increment. For instance, the $\overline{r^{\max}-r^{\min}}$ of CMT1 can be reduced to 3/4 of its $\overline{\overline{r^{\max}-r^{\min}}}$ by increasing 2.31% of its original distance. In order to reduce the $\overline{r^{\max}-r^{\min}}$ to 1/2 of $\overline{\overline{r^{\max}-r^{\min}}}$, the travel distance may need to be extended to as big as 35.92% and 18.46% for CMT11 and CMT12, respectively. Moreover, considering the value of ε equals to the value of 1/4 of $\overline{\overline{r^{\max}-r^{\min}}}$ requires much bigger ν to achieve a solution, and the travel distance extension is larger than anticipated. It is noted that considering the value of ε equals to 1/4 of $\overline{\overline{r^{\max}-r^{\min}}}$, the solutions for CMT5 and CMT12 cannot be figured out here.

Hypothetically, a much more condensed max route should help reduce the distance difference between the maximum and the minimum route, which means better-balanced routes. In light of this premise, the objective function is revised as Equation (23) by adding the equity measurement min-max, e.g. r^{max}, into the objective function.

$$\min \quad \sum_{i\in V}\sum_{j\in V, i\neq j}\sum_{k\in K} c_{ij}x_{ij}^{k} + \nu\zeta + \mu r^{\max} \tag{23}$$

The parameter μ in the last term of the objective function (23) is the weight of the max route. This term is anticipated to compact the max route so that all routes reach balance. Equation (23) is a general form in which Equation (22) is the case when μ=0. The benchmark dataset is solved under the scenarios that μ =1, 2, and 3 to verify the abovementioned hypothesis. The output results are listed in Table 3. It can be observed that we can find some better solutions that dominated the solutions while μ=0. However, the results are tangled, so it is difficult to distinguish the trend.

Table 2. The results of ASO on the route balancing problem

			ε=(3/4) $\overline{\overline{r^{\max}-r^{\min}}}$			
Instance	s	ε	TD	$\overline{r^{\max}-r^{\min}}$	ν	dir
CMT1	524.61	15.05	536.74	11.17	2	2.31%
CMT2	840.94	58.12	861.39	30.20	1	2.43%
CMT3	827.39	50.66	908.64	24.84	2	9.82%
CMT4	1055.00	67.57	1068.15	52.59	1	1.25%
CMT5	1401.94	87.89	1416.88	41.81	1	1.07%
CMT11	1043.65	110.00	1271.95	107.91	2	21.88%
CMT12	819.56	70.07	931.44	46.70	4	13.65%
			ε=(1/2) $\overline{\overline{r^{\max}-r^{\min}}}$			
Instance	s	ε	TD	$\overline{r^{\max}-r^{\min}}$	ν	dir
CMT1	524.61	10.04	546.00	8.42	4	4.08%
CMT2	840.94	38.75	861.39	30.20	1	2.43%
CMT3	827.39	33.78	908.64	24.84	2	9.82%
CMT4	1055.00	45.05	1108.14	39.09	2	5.04%
CMT5	1401.94	58.60	1416.88	41.81	1	1.07%
CMT11	1043.65	73.34	1418.51	42.50	5	35.92%
CMT12	819.56	46.72	970.85	40.46	5	18.46%
			ε=(1/4) $\overline{\overline{r^{\max}-r^{\min}}}$			
Instance	s	ε	TD	$\overline{r^{\max}-r^{\min}}$	ν	dir
CMT1	524.61	5.02	550.15	4.85	12	4.87%
CMT2	840.94	19.37	934.69	19.14	15	11.15%
CMT3	827.39	16.89	887.61	16.73	5	7.28%
CMT4	1055.00	22.52	1193.97	22.42	20	13.17%
CMT5	1401.94	29.30	-	-	-	-
CMT11	1043.65	36.67	1440.60	26.86	10	38.03%
CMT12	819.56	23.36	-	-	-	-

Note: s: CVPR solution achieved by ASO; $\overline{\overline{r^{\max}-r^{\min}}}$: r^{max} - r^{min} of CVPR solution achieved by ASO; ε: the threshold values of equity measurement; TD: total travel distance of VRPRB; $\overline{r^{\max}-r^{\min}}$: r^{max} - r^{min} of the solution of VRPRB; ν: the weight of equity measurement when the solution is achieved; dir: distance increment ratio, $\frac{\mathrm{TD}-\mathrm{s}}{\mathrm{s}} \times 100\%$.

Table 3. Results of ASO on the balance problem considering different μ

Instance		CMT1				CMT2				CMT3				CMT4				CMT5				CMT11				CMT12			
μ		0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
3/4	$\varepsilon=(3/4)\,\overline{\overline{r^{\max}-r^{\min}}}$	15.05				58.12				50.66				67.57				87.89				110.00				70.07			
	$\overline{r^{\max}-r^{\min}}$	11.17	11.17	14.52	14.52	30.20	40.02	35.86	33.24	24.84	37.65	28.86	32.61	52.59	43.90	39.79	43.90	41.81	37.89	51.02	41.81	107.91	32.28	25.50	77.54	46.70	45.91	66.45	67.97
	v	2	3	1	1	1	2	1	5	2	1	1	1	1	1	4	2	1	3	4	1	2	3	16	2	4	2	4	3
	TD	536.74	536.74	**531.90**	**531.90**	861.39	876.69	878.08	890.28	908.64	**847.76**	**852.31**	**862.27**	1068.15	1095.21	1148.39	1093.29	1416.88	1424.96	1520.38	1416.88	1271.95	1472.43	1430.71	**1228.21**	931.44	988.06	935.63	**845.85**
	ratio (%)	0	0	**-0.90**	**-0.90**	0	1.78	1.94	3.35	0	**-6.70**	**-6.20**	**-5.10**	0	2.5	7.51	2.35	0	0.57	7.30	0.00	0	15.76	12.48	**-3.44**	0	6.08	0.45	**-9.19**
1/2	$\varepsilon=(1/2)\,\overline{\overline{r^{\max}-r^{\min}}}$	10.04				38.75				33.78				45.05				58.60				73.34				46.72			
	$\overline{r^{\max}-r^{\min}}$	8.42	8.83	6.38	7.63	30.20	34.56	35.86	33.24	24.84	22.94	23.05	23.91	39.09	43.90	39.79	43.90	41.81	37.89	51.02	41.81	42.50	32.28	25.50	29.16	40.46	41.03	27.37	42.05
	v	4	6	8	5	1	1	1	5	2	26	4	2	2	1	4	2	1	3	4	1	5	3	16	20	5	1	2	1
	TD	546.00	547.78	549.07	549.67	861.39	864.41	878.08	890.28	908.64	970.12	924.4	**874.05**	1108.14	**1095.21**	1148.39	**1093.29**	1416.88	1424.96	1520.38	1416.88	1418.51	1472.43	1430.71	**1416.69**	970.85	1032.71	1095.47	**848.12**
	ratio (%)	0	0.33	1.37	0.40	0	0.35	1.94	3.35	0	6.77	1.73	**-3.81**	0	**-1.17**	3.63	**-1.34**	0	0.57	7.30	0.00	0	3.80	0.86	**-0.13**	0	6.37	12.84	**-12.64**
1/4	$\varepsilon=(1/4)\,\overline{\overline{r^{\max}-r^{\min}}}$	5.02				19.37				16.89				22.52				29.30				36.67				23.36			
	$\overline{r^{\max}-r^{\min}}$	4.85	4.92	4.85	4.92	19.14	18.74	17.76	18.38	16.73	16.18	16.63	16.84	22.42	21.93	22.22	21.93	-	25.90	29.21	-	26.86	32.28	25.50	29.16	-	-	21.58	20.50
	v	12	8	14	19	15	7	15	26	5	3	7	12	20	8	25	28	-	28	5	-	10	3	16	20	-	-	25	9
	TD	550.15	559.47	550.15	559.47	934.69	**917.30**	**929.80**	**947.76**	887.61	**881.21**	897.27	943.93	1193.97	**1155.04**	1224.03	**1186.24**	-	1726.47	1638.44	-	1440.60	1472.43	**1430.71**	**1416.69**	-	-	1101.05	893.71
	ratio (%)	0	1.69	0	1.69	0	**-2.09**	**-7.21**	**-3.97**	0	**-0.72**	1.09	6.35	0	**-6.65**	2.52	**-0.65**	-	-	-	-	0	2.21	**-0.69**	**-1.66**	-	-	-	-

Note: Ratio = {(TD – TD_{u0}) / TD_{u0}}*100%; TD_{u0}: TD while μ=0.

Table 4 lists the smallest (r^{max} - r^{min}) and other affiliated information found during the parameter analysis. The process can be observed to reduce the (r^{max} - r^{min}) to a pretty little value with increasing total travel distance. It can also be discerned that the minimal (r^{max} - r^{min}) is not necessary to occur when μ=3. That means bigger values of μ and v do not guarantee better balance routes but have a higher probability of obtaining one.

From the above experiments, we can find that while the threshold ε is getting bigger, indicating the balance restriction is more relaxing. That implies the travel cost is getting bigger as well; on the other hand, the shifts for workers are more balanced. It is a trade-off between cost and workers' satisfaction. According to Figure 2, the shifts can be almost the same among the workers in the case with a higher cost (586.84) for CMT1. It is possibly more beneficial to choose the costly case instead of the inexpensive cases because the workers' satisfaction is high. The managers should determine the dispatching scheme between low-cost and possibly unsatisfied workers.

5.3: the relationship between r^{max} - r^{min}, v and μ

We further apply the regression analysis to examine how (r^{max} - r^{min}) varies with v and μ since it is challenging to conclude their relation by plotting results. Equation (24) represents our regression model, where β_0 to β_4 are parameters and ε is the error term. After graphing our previous results, we assume a semi-log functional form; that is, we transform r^{max} - r^{min} to ln(r^{max} - r^{min}) as our dependent variable values. The independent variables include v and a set of dummy variables, Dummy1, Dummy2, and Dum-

Table 4. Solutions for minimizing max-min

	CMT1	CMT2	CMT3	CMT4	CMT5	CMT11	CMT12
μ	2	2	0	1	3	1	3
v	34	48	31	25	45	5	18
s	524.61	840.94	827.39	1055	1401.94	1043.65	819.56
TD	586.84	946.51	978.89	1196.94	1594.72	1470.19	957.17
dir	11.86%	12.55%	18.31%	13.45%	13.75%	40.87%	16.79%
$\overline{\overline{r^{\max}-r^{\min}}}$	20.07	77.49	67.55	90.09	117.19	146.67	93.43
$\overline{r^{\max}-r^{\min}}$	0.28	4.43	1.65	9.51	24.51	2.86	3.38
ratio	1.40%	5.72%	2.44%	10.56%	20.91%	1.95%	3.62%

Note: ratio= ($\overline{r^{\max}-r^{\min}}$ / $\overline{\overline{r^{\max}-r^{\min}}}$)*100%

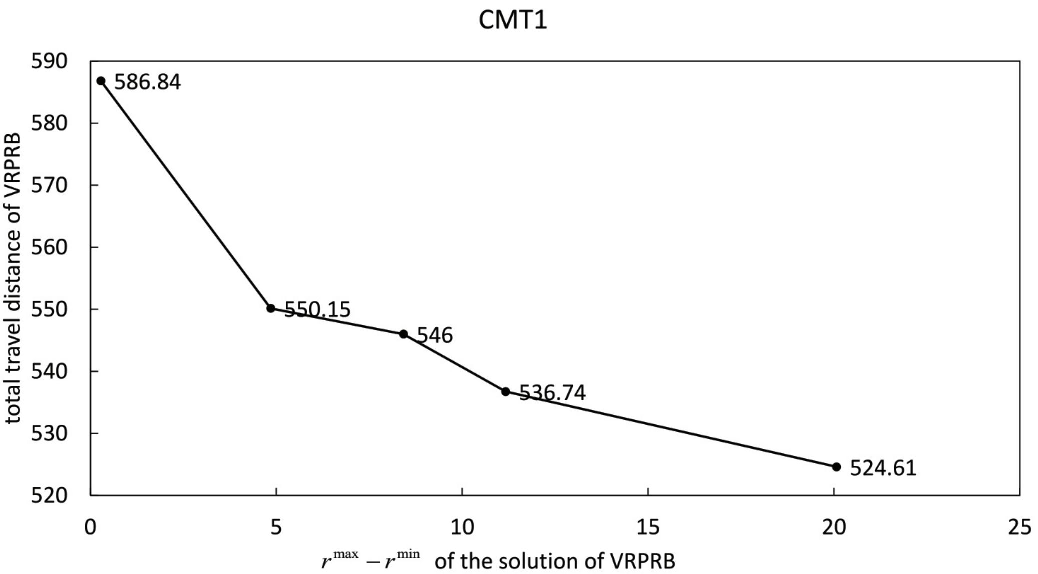

Figure 2. Relationships between route distance and the (r^{max} - r^{min}) of CMT1

my3, for μ equals 1, 2, and 3, respectively, to reveal intercept deviation from the base, as μ equals 0. This model assumes a fixed slope of ν, but allows intercept to vary with different values of μ. If the estimated coefficients are negative/positive, this shows that for any given ν, a non-zero μ will generate a smaller/larger (r^{max} - r^{min}). Table 5 shows our estimation results. The estimated coefficient of ν is negative and significant, meaning that as ν gets bigger, (r^{max} - r^{min}) gets smaller, and our previous solutions statistically support this relation. Coefficients for the Dummy1, Dummy 2, and Dummy 3 are all negative, indicating that the use of parameter μ will reduce the value of (r^{max} - r^{min}) comparing to the base when μ equals 0; however, the hypothesis test shows the reduction of objective function value by non-zero μ values is not statistically significant.

$$\ln r^{\max} - r^{\min} = \beta_0 + \beta_1 Dummy_1 + \beta_2 Dummy_2 + \beta_3 Dummy_3 + \beta_4 \nu + \varepsilon \tag{24}$$

Although Table 5 shows an insignificant influence of μ to (r^{max} - r^{min}) for given ν, it is possible to observe a clear impact of μ if we apply a more flexible functional form, allowing the relationship to change in a higher ν situation. We use ν =20 as a cutting point. High is a dummy variable to indicate whether ν is greater than or equal to 20. Therefore, the variable High will equal 1 if $\nu \geq 20$ and 0 otherwise. We allow intercepts for given μ and the slope of ν to be different if $\nu \geq 20$ by including intersection terms. The estimation results in Table 6 show that the slope of ν will be flatter when $\nu \geq 20$ since the slope is from -0.0757 change to -0.0093, indicating a statistically significant but weakened influence when ν becomes higher. We observe that a non-zero μ will, on average, produce a better solution compared to μ =0 case when $\nu \geq 20$ and is statistically supported when μ equals 2 and 3.

6. Conclusion and future research

This study proposed a hybrid algorithm, ASO, to solve the Vehicle Routing Problem with route balancing. The solving process has been applied to the benchmark instances CMT1-5, 11, and 12 to examine the performance. We solved the VRPRB by combining the total travel distance and the equity measurements as the objective function so that the travel distance and equity measurements can be optimized simultaneously. The parameters μ and ν are used to adjust the weights of the equity measurements in the objective function. The output results show that the proposed algorithm can provide promising solutions.

The goal of this research is to figure out the balance point between the total travel distance and the range, i.e., r^{max} - r^{min}. A specific amount of solution quality must be sacrificed to obtain better balance routes. Other researchers might concentrate on minimizing the (r^{max} - r^{min}) such as Borgulya [19]. The (r^{max} - r^{min}) can be as small as 0.03, taking CMT1 as an example, but the total travel distance is more than three times of best solution. On the contrary, this proposed solving process can diminish the equity measurement to 0.28 (CMT1); however, the travel distance increment ratio is just 11.13%. Thus, it is undoubtedly a trade-off issue for management to determine how much cost is allowed to exchange for balanced routes.

Using the total travel time as the objective function may be more realistic than the total travel distance while handling the VRPRB. Thus, it is worth modifying the objective function to be the total working time in future works. Also, other equity measurements, such as the average sum of relative imbalance, can be used to estimate the route balance. We can add these measurements to the objective function

Table 5. Examine the relation of (r^{max} - r^{min}) and ν for various μ

Variable	Coefficient	Standard error	t	p-value
ν	-.0523	.004622	-11.31	0.000
Dummy1	-.1000	.114025	-0.88	0.383
Dummy2	-.0532	.106288	-0.50	0.618
Dummy3	-.0993	.110694	-0.90	0.372
intercept	2.3898	.102481	23.32	0.000
R^2	0.5664			
F	33.02			
Prob > F	0.0000			

to see whether better-balanced routes can be found in the future. Furthermore, it is interesting to know how the proposed method yields while tackling more complex VRP instances. For example, balancing the VRPs with different situations, such as time windows, pickup & delivery, could be good research directions.

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References

[1] M. M. Solomon, “Algorithms for the vehicle routing and scheduling problems with time window constraints,” Oper. Res., vol. 35, no. 2, pp. 166–324, 1987, doi: 10.1287/opre.35.2.254.

[2] E. Choi, and D. W. Tcha, “A column generation approach to the heterogeneous fleet vehicle routing problem,” Comput. Oper. Res., vol. 34, no. 7, pp. 2080–2095, 2007, doi: 10.1016/j.cor.2005.08.002.

[3] M. Yousefikhoshbakht, F. Didehvar, and F. Rahmati, “Solving the heterogeneous fixed fleet open vehicle routing problem by a combined metaheuristic algorithm,” Int. J. Prod. Res., vol. 52, no. 9, pp. 2565–2575, 2014, doi: 10.1080/00207543.2013.855337.

[4] M. Polacek, R. F. Hartl, K. Doerner, and M. Reimann, “A variable neighborhood search for the multi depot vehicle routing problem with time windows,” J. Heuristics, vol. 10, pp. 613–627, 2004, doi: 10.1007/s10732-005-5432-5.

[5] R. Dondo, and J. Cerda, “A cluster-based optimization approach for the multi-depot heterogeneous fleet vehicle routing problem with time windows,” Eur. J. Oper. Res., vol. 176, pp. 1478–1507, 2007, doi: 10.1016/j.ejor.2004.07.077.

[6] B. Crevier, J. F., Cordeau, and G. Laporte, “The multi-depot vehicle routing problem with inter-depot routes,” Eur. J. Oper. Res., vol. 176, pp. 756–773, 2007, doi: 10.1016/j.ejor.2005.08.015.

[7] M. Ostermeier, S. Martins, P. Amorim, and A. Hübner, “Loading constraints for a multi-compartment vehicle routing problem,” OR Spectrum, vol. 40, pp. 997–1027, 2018, doi: 10.1007/s00291-018-0524-4.

[8] M. Reed, A. Yiannakou, and R. Evering, “An ant colony algorithm for the multi-compartment vehicle routing problem,” Appl. Soft Comput., vol. 15, pp. 169–176, 2014, doi: 10.1016/j.asoc.2013.10.017.

[9] H. Hernandez-Perez, and J. J. Salazar-Gonzalez, “Heuristics for the one commodity pickup-and-delivery travelling salesman problem,” Transport. Sci., vol. 38, pp. 245–255, 2004, doi: 10.1287/trsc.1030.0086.

[10] H. Hernandez-Perez, I., Rodrguez-Martn, and J. J. Salazar-Gonzalez, “A hybrid heuristic approach for the multi-commodity pickup-and-delivery traveling salesman problem,” Eur. J. Oper. Res., vol. 251, pp. 44–52, 2016, doi: 10.1016/j.ejor.2015.10.053.

[11] O. Belgin, I. Karaoglan,, and F. Altiparmak, “Two-echelon vehicle routing problem with simultaneous pickup and delivery: Mathematical model and heuristic approach,” Comput.Ind. Eng., vol. 115, pp. 1–16, 2018, doi: 10.1016/j.cie.2017.10.032.

[12] Y. H. Huang, C. A. Blazquez, S. H. Huang, and G. Paredes-Belmar, “Solving the Feeder Vehicle Routing Problem using ant colony optimization,” Comput. Ind. Eng., vol. 127, pp. 520–535, 2019, doi: 10.1016/j.cie.2018.10.037.

[13] H. Park, and S. Jin, “Electric vehicle routing problem with heterogeneous vehicles and partial charge,” Int. J. Ind. Eng. Manage., vol. 11, no.4, pp. 215–225, 2020, doi: 10.24867/IJIEM-2020-4-266.

[14] A. L. Erera, J. C. Morales, and M. Savelsbergh, “The vehicle routing problem with stochastic demand and duration constraints,” Transport. Sci., vol. 44, no. 4, pp. 474–492, 2010, doi: 10.1287/trsc.1100.0324.

[15] S. H. Huang, and P. C. Lin, “A modified ant colony optimization algorithm for multi-item inventory routing problems with demand uncertainty,” Transport. Res. E, vol. 46, pp. 598–611, 2010, doi: 10.1016/j.tre.2010.01.006.

[16] G. Laporte, F. V. Louveaux, and L. van Hamme, “An integer L-Shaped algorithm for the capacitated vehicle routing problem with stochastic demands,” Oper. Res., vol. 50, no. 3, pp. 415–423, 2002, doi: 10.1287/opre.50.3.415.7751.

[17] W. Rei, M. Gendreau, and P. “A hybrid Monte Carlo local branching algorithm for the single vehicle routing problem with stochastic demands,” Transport. Sci., vol. 44, no. 1, pp. 136–146, 2010, doi: 10.1287/trsc.1090.0295.

[18] L. C. Lu, and T. W. Yue, “Mission-oriented ant-team ACO for min-max MTSP,” Appl. Soft Comput., vol. 76, pp. 436–444, 2019, doi: 10.1016/j.asoc.2018.11.048.

[19] I. Borgulya, “An algorithm for the capacitated vehicle routing problem with route balancing,” Cent. Eur. J. Oper. Res., vol. 16, no. 4, pp. 331–343, 2008, doi: 10.1007/s10100-008-0062-2.

[20] E. E. Halvorsen-Weare, and M. W. P. Savelsbergh, “The bi-objective mixed capacitated general routing problem with different route balance criteria,” Eur. J. Oper. Res., vol. 251, no. 2, pp. 451–465, 2016, doi: 10.1016/j.ejor.2015.11.024.

[21] Z. Zhang, Y. Sun, H. Xie, Y. Teng, and J. Wang, “GMMA: GPU-based multi-objective memetic algorithms for vehicle routing problem with route balancing,”. Appl. Intell., vol. 49, pp. 63–78, 2019, doi: 10.1007/s10489-018-1210-6.

[22] J. Oyola, and A. Lokketangen, “GRASP-ASP: An algorithm for the CVRP with route balancing,” J. Heuristics, vol 20, pp. 361–382, 2014, doi: 10.1007/s10732-014-9251-4.

[23] N. Jozefowiez, F. Semet, and E.G. Talbi, “An evolutionary algorithm for the vehicle routing problem with route balancing,” Eur. J.Oper. Res., vol. 195, pp. 761–769, 2009, doi: 10.1016/j.ejor.2007.06.065.

[24] T. Keskinturk, and M. B. Yildirim, “A genetic algorithm metaheuristic for bakery distribution vehicle routing problem with load balancing,” in Innovations in Intelligent Systems and Applications (INISTA), 2011, doi: 10.1109/INISTA.2011.5946077.

[25] J. Lozano, L. -C. González-Gurrola, E. Rodríguez-Tello and P. Lacomme, "A Statistical Comparison of Objective Functions for the Vehicle Routing Problem with Route

Balancing," in 2016 Fifteenth Mexican International Conference on Artificial Intelligence (MICAI), Cancun, Mexico, 2016, pp. 130-135, doi: 10.1109/MICAI-2016.2016.00028.

[26] L. I. Jingjing, Y. Fang, and N. Tang, "A cluster-based optimization framework for vehicle routing problem with workload balance," Comput. Ind. Eng., vol. 169, 108221, 2022, doi: 10.1016/j.cie.2022.108221.

[27] R. Linfati, F. Yáñez-Concha, and J. W. Escobar, "Mathematical models for the vehicle routing problem by considering balancing load and customer compactness," Sustainability, vol. 14, no. 19, 12937, 2022, doi: 10.3390/su141912937.

[28] B. L. Golden, G. Laporte, and E. D. Taillard, "An adaptive memory heuristic for a class of vehicle routing problems with minmax objective," Comput. Oper. Res., vol. 24, no. 5, pp. 445–452, 1997, doi: 10.1016/S0305-0548(96)00065-2.

[29] L. Bertazzi, B. Golden, and X. Wang, "Min-max vs. min-sum vehicle routing: a worst-case analysis," Eur. J. Oper. Res., vol. 240, pp. 372–381, 2015, doi: 10.1016/j.ejor.2014.07.025.

[30] E. Yakici, "A heuristic approach for solving a rich min-max vehicle routing problem with mixed fleet and mixed demand," Comput. Ind. Eng., vol. 109, pp. 288–294, 2017, https://doi.org/10.1016/j.cie.2017.05.001.

[31] M. Rabbani, M. Akbarpour, M. Hosseini, and H. Farrokhi-Asl, "A multi-depot vehicle routing problem with time windows and load balancing: a real world application," Int. J. Supply and Operations Managenent, vol. 8, no. 3, pp. 347–369, 2021, doi: 10.22034/IJSOM.2021.3.7.

[32] K. V. Narasimha, E. Kivelevitch, B. Sharma, and M. Kumar, "An ant colony optimization technique for solving min-max multi-depot vehicle routing problem," Swarm Evol. Comput., vol. 13, pp. 63–73, 2013, doi: 10.1016/j.swevo.2013.05.005.

[33] X. Wang, B. Golden, E. Wasil, and R. Zhang, "The min-max split delivery multi-depot vehicle routing problem with minimum service time requirement," Comput. Oper. Res., vol. 71, pp. 110–126, 2016, doi: 10.1016/j.cor.2016.01.008.

[34] Y. Wang, L. Zhao, M. Savelsbergh, and S. Wu, "Multi-period workload balancing in last-mile urban delivery," Transport. Sci., vol. 56, no. 5, pp. 1348–1368, 2022, doi: 10.1287/trsc.2022.1132.

[35] J. Kennedy and R. Eberhart, "Particle swarm optimization," in Proceedings of ICNN'95 - International Conference on Neural Networks, Perth, WA, Australia, 1995, pp. 1942-1948 vol.4, doi: 10.1109/ICNN.1995.488968.

[36] M. Dorigo, and L.M. Gambardella, "Ant colony system: a cooperative learning approach to the traveling salesman problem,". IEEE T. Evolut. Comput., vol. 1, no. 1, pp. 53–66, 1997, doi: 10.1109/4235.585892.

[37] N. Christofides, A. Mingozzi, and P. Toth, "The vehicle routing problem" in Combinatorial Optimization, N. Christofides, A. Mingozzi, P. Toth, and C. Sandi, Eds. Chichester, USA: John Wiley & Sons, 1979, pp. 313–338.