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Emerging Markets Unidirectional Sensitivity Coefficient as an Indicator in Portfolio Investors' Decision Making

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Abstract

This paper examines linear correlations between six emerging European stock market indices and the world's most significant index Standard & Poor's (S&P) 500. Using a unique dataset within three years of data on the indices from the markets in Bulgaria, Croatia, Hungary, Romania, Serbia and Slovenia we compare movements of the indices, find out the influences on their regularity in day-to-day movements and calculate their unidirectional correlation. We introduce movement relative sensitivity to changes in S&P 500 values for each regional index, in order to better describe unidirectional changes. Results for unidirectional correlation and movement relative sensitivity show that these two indicators can be important factor in an investor decision making process. Furthermore, we explain how these two indicators can be used as practical tool in algorithmic trading system development. Finally, we show that average unidirectional sensitivity coefficient can be used as volatility indicator.

Key words: *Unidirectional sensitivity coefficient, Emerging markets, Market correlations, Market co-movement*

1. INTRODUCTION

Since the beginning of transition period, the emerging markets in Europe are becoming more and more attractive for international and domestic investors willing to diversify their portfolio investments by investing in local stock markets [1]. Some authors state that international investors do have a major influence on emerging markets in focus [2]. Investors take into consideration rapid economic development of those countries, potential high returns, portfolio diversification potentials and constant reforms and liberalizations on these capital markets [3]. Volume of trading and market capitalization of major stock market exchanges in emerging markets point out the situation of financial markets in general. Due to increased globalization of world economy, markets react very quickly to the information revealed in the prices from other markets [4]. The aim of this paper is to examine market co-movements through the influence of world's most important index, S&P 500, on regional emerging stock market indices movements.

Due to different time zones, we will compare previous trading day closing values of S&P 500 to six emerging markets indices current day closing values. Based on results of linear correlation, we will propose new indicators that can be used as an additional criteria in a decision making process. By introducing unidirectional correlation coefficient, we want to describe how benchmark index S&P 500 and given market index simultaneously increase or decrease. Due to higher volatility of observed markets compared to S&P 500, we will examine how much faster an index grows (falls) when triggered by increase (decrease) in benchmark value. Results will provide quantification of unidirectional behaviour of benchmark S&P 500 index and regional stock market indices. We will explain how unidirectional correlation approach can be applied in equity investing.

The paper is organized as follows. In Section 2 we give theoretical background and explain the term of market correlation. Section 3 explains data used for research as well as the criteria for sample extraction for countries of interest.

Also, the used methodology is presented within the same section. In Section 4 we present the results and discussion. Finally, concluding remarks and suggestions for future research are stated in Section 5. We use MATLAB 6.0. program environmet for data processing in our work.

2. THEORETICAL BACKGROUND

Portfolio investment decision making is a process which faces serious risk and market growth potential analysis. While making their decision where to invest and how to diversify their investment portfolio, investors around the globe consider various risk perspectives. Risk and return models in finance take into consideration investor risk aversion, information uncertainty and perceptions of macroeconomic risk [5].

Studying stock market contagion during financial crises, some authors have proposed a hypothesis that international participants ignore fundamental international economic information and simply focus on price movements in other countries, particularly the United States [6]. They concluded that it is difficult to explain the international correlation structure with macroeconomic variables, while significant co-movements among world equity markets are observed. Their work indicated that there are strong regional transmission effects. In their work, other authors have shown that correlation coefficients are conditional upon market volatility [7]. Their study confirms the importance of simultaneously giving consideration to return and volatility co-movements when studying the intensity of return transmissions.

Recent research on world stock index volatility in relation to correlation with US market, showed interdependence in values. Even stronger evidence of rising correlations is reported on a regional basis [8, 9].

3. DATA AND METHODOLOGY

3.1 Data set description

The data used for correlation calculations in this paper are accessible from websites of emerging markets stock exchanges and Standard & Poor’s website. We conduct our experiment on six Central and South Eastern European countries, where the sample is within three years period of time, between January 2007 and December 2009.

We use daily closing prices of the following country major stock market indices: BELEX LINE (Belgrade Stock Exchange), BET (Bucharest Stock Exchange), BUX (Budapest Stock Exchange), CROBEX (Zagreb Stock Exchange), SBI 20 (Ljubljana Stock Exchange) and SOFIX (Bulgarian Stock Exchange - Sofia). In order to have relevant sample of indices values, we compared trading days for all stock markets in focus from January 2007 until December 2009. We consider only overlapping trading days of each regional index with S&P 500 within three years period of time. Due to difference in working calendars (public holidays), few trading days were not taken into consideration.

The final sample size varies from 719 and 737 closing prices for each pair of six regional indices with S&P 500 (Table 1).

Table 1. Sample size by indices pairs

Indices pairs	Sample size n
BELEX LINE - S&P 500	737
BUX - S&P 500	725
BET - S&P 500	729
CROBEX - S&P 500	724
SBI 20 - S&P 500	728
SOFIX - S&P 500	719

Criteria for sample extraction took into consideration different time zones and the fact that we want to observe S&P 500 daily price increase (decrease) as a correlation benchmark. In other words, we will know daily price movement of S&P 500 only when US trading resumes for a day, and therefore, we calculated correlation using current day closing value of regional indices with previous trading day closing value of S&P 500 index. Table 2 shows the trading hours on stock markets in focus, according to Greenwich Mean Time (GMT). This approach in extracting data set creates significant opportunity to observe and compare values with significant time lag. This way, we can examine changes in regional indices as a result of S&P 500 influence without additional conditions.

Table 2. Trading hours on stock markets in focus

Index	Trading hours refereed to GMT time	
	from	*to*
S&P 500	14:30	21:00
BUX	08:00	15:30
BET	08:00	14:25
BELEX LINE	09:00	11:00
CROBEX	09:00	15:00
SBI 20	08:30	12:00
SOFIX	08:00	12:45

3.2 Mathematical background and methodology

We observe two samples $x=(x_1, x_2, ..., x_n)$ and $y=(y_1, y_2, ..., y_n)$ with size n, where $n \in N$, $x_i \in R^+$, $y_i \in R^+$, $i=1, 2, ..., n$.

Mathematical formulas used in further analysis, are listed below:

1. Mean value

$$\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i \; ; \quad \bar{y}=\frac{1}{n}\sum_{i=1}^{n} y_i \tag{1}$$

2. Linear correlation coefficient

$$r_L^{xy} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}} \quad (2)$$

3. Absolute change of the sample on the position *i*

$$\Delta_i^x = x_i - x_{i-1} \ , \quad (3)$$
$$\Delta_i^y = y_i - y_{i-1} \ ,$$

where $i = 2, 3, ..., n$.

4. Contribution of the sample on the position *i*

$$\delta_i^x = \frac{x_i}{x_{i-1}} \ , \quad (4)$$
$$\delta_i^y = \frac{y_i}{y_{i-1}} \ ,$$

where $i = 2, 3, ..., n$.

5. Relative change of the sample on the position *i*

$$\rho_i^x = \frac{\Delta_i^x}{x_{i-1}} = \frac{x_i - x_{i-1}}{x_{i-1}} = \delta_i^x - 1 \ , \quad (5)$$
$$\rho_i^y = \frac{\Delta_i^y}{y_{i-1}} = \frac{y_i - y_{i-1}}{y_{i-1}} = \delta_i^y - 1 \ ,$$

where $i = 2, 3, ..., n$.

6. For samples x and y we form vectors of absolute changes, $(\Delta_2^x, \Delta_3^x, ..., \Delta_n^x)$ and $(\Delta_2^y, \Delta_3^y, ..., \Delta_n^y)$, with size $n-1$. Further more, we define following parameters for those vectors:

$$^{+}m^{xy} = \sum_{i=2}^{n} 1, \quad \Delta_i^x > 0 \wedge \Delta_i^y > 0 \ , \quad (6)$$
$$^{-}m^{xy} = \sum_{i=2}^{n} 1, \quad \Delta_i^x < 0 \wedge \Delta_i^y < 0 \ ,$$
$$m^{xy} = {}^{+}m^{xy} + {}^{-}m^{xy}$$

Using parameter m^{xy} we define unidirectional correlation coefficient r_U^{xy} of the samples x and y .

DEFINITION 1.

Unidirectional correlation coefficient r_U^{xy} of the samples x and y is:

$$r_U^{xy} = \frac{m^{xy}}{n-1} \quad (7)$$

In the first row of Figure 1 are samples *x* and *y* with sample size *n* = 7. For each position $i = 1, 2, 3, 4, 5, 6, 7$, in second row, we graphically present absolute changes Δ^x and Δ^y .

Finally, in legend bellow, we count number of hits for $^{+}m^{xy}$ and $^{-}m^{xy}$. In this example, we get that $^{+}m^{xy} = 2$ and $^{-}m^{xy} = 2$ and unidirectional coefficient, based on (7), is $r_U^{xy} = \frac{4}{7-1} = 0.6667$.

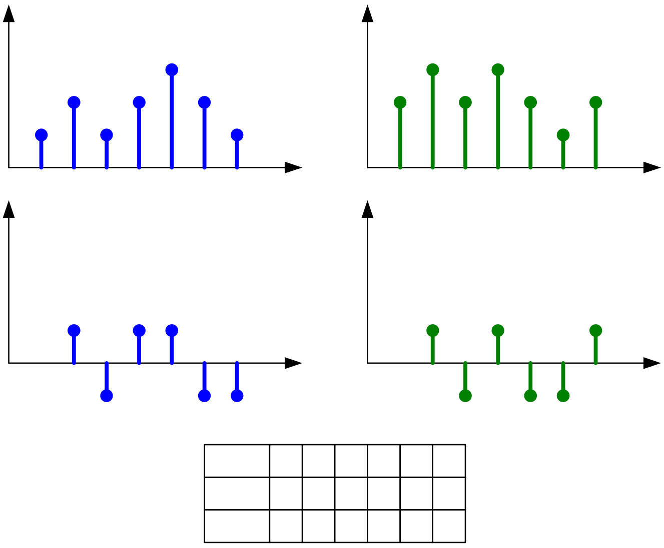

Figure 1. Detecting $^{+}m^{xy}$ and $^{-}m^{xy}$

Now, we define unidirectional sensitivity coefficient ψ_i^{xy} of the sample x in relation to sample y (benchmark sample) on the position i, and then, using mathematical theorems (9) and (10), we examine its characteristics (theorems proofs are in Appendix 1).

DEFINITION 2.

Unidirectional sensitivity of sample x in relation to sample y on the position i is:

$$\psi_i^{xy} = \frac{\rho_i^x - \rho_i^y}{\rho_i^y} \ , \quad (8)$$
$$i = 2, 3, ..., n \wedge \left((\Delta_i^x > 0 \wedge \Delta_i^y > 0) \vee (\Delta_i^x < 0 \wedge \Delta_i^y < 0) \right)$$

THEOREM 1.

Let x and y be samples with size n , then:

$$\psi_i^{xy} \neq 0 \quad \Leftrightarrow \quad \psi_i^{xy} \neq \psi_i^{yx} \quad (9)$$

THEOREM 2.

Let x_1, x_2 and y be samples with size n , then:

$$\mathrm{sgn}\left(\psi_i^{x_1 y} - \psi_i^{x_2 y}\right) = \mathrm{sgn}\left(\delta_i^{x_1} - \delta_i^{x_2}\right) \quad (10)$$

Unidirectional sensitivity coefficient ψ_i^{xy} is practically applicable especially for samples with high linear correlation coefficient (we propose $r_L^{xy} > 0.9$). In order to put in relation general influence of benchmark sample *y* to another sample *x*, we will use average unidirectional sensitivity coefficient $\overline{\psi^{xy}}$ in further presentation.

Note that we can calculate ψ_i^{xy} only on the correlated sample positions i, $i = 2, 3, ..., n$ (when both rise or fall on the same position). The number of correlated sample positions is parameter m^{xy}.

DEFINITION 3.

Average unidirectional sensitivity coefficient of the sample x in relation to sample y is:

$$\overline{\psi^{xy}} = \frac{1}{m^{xy}} \sum_{i=2}^{n} \psi_i^{xy}, \left(\left(\Delta_i^x > 0 \wedge \Delta_i^y > 0 \right) \vee \left(\Delta_i^x < 0 \wedge \Delta_i^y < 0 \right) \right) \quad (11)$$

Using following example, we can explain practical contribution of (6). If $\overline{\psi^{x_1 y}} > \overline{\psi^{x_2 y}}$ for samples x_1, x_2 and y with size n, then we can expect that sample x_1 will have more intensive movement (either rise or fall) in comparison to sample x_2, if change in sample y occurs.

4. RESULTS AND DISCUSSION

4.1 Linear correlations analysis

Results presented in Table 3 show linear correlation coefficient r_L calculated through mathematical formula for linear correlation (2). Results are sorted in descending order.

Table 3. Linear correlation coefficient r_L for each indices pair

Indices pairs	Correlation coefficient r_L
BET - S&P 500	0.9725
BUX - S&P 500	0.9568
SOFIX - S&P 500	0.9463
BELEX LINE - S&P 500	0.9448
CROBEX - S&P 500	0.9374
SBI 20 - S&P 500	0.9011

There is a strong evidence for high linear correlation in case of every considered indices pair. These are expected values, according to theory, for a sample of regional emerging markets.

Quality of this result is that, we have a window of opportunity to observe two correlated signals with significant time delay. However, linear correlation describes general, long term dependence of data and can not have practical implementation.

In order to further examine co-movement of S&P 500 and regional indices, we introduced unidirectional correlation.

4.2 Indices' unidirectional correlation

Results presented in Table 4 show unidirectional correlation coefficient r_U calculated through mathematical formula for unidirectional correlation (7).

Table 4. Unidirectional correlation

Indices pairs	Parameters			
	^{+}m	^{-}m	m	r_U
BET - S&P 500	229	208	437	0.6003
BUX - S&P 500	215	191	406	0.5608
SOFIX - S&P 500	234	220	454	0.6323
BELEX LINE - S&P 500	195	220	415	0.5639
CROBEX - S&P 500	217	193	410	0.5671
SBI 20 - S&P 500	230	198	428	0.5887

Unidirectional correlation exists when a regional stock market has positive (negative) price change as a result of increase (decrease) in previous trading day S&P 500 value. Total number of hits, m is a sum of positive co-movements ^{+}m and negative co-movements ^{-}m. Unidirectional correlation coefficient r_U presents probability that a regional index and benchmark (S&P 500) will move in a same direction, with significant time delay.

Value of unidirectional correlation for an index contributes to prediction possibility, unlike linear correlation, where we can only see a general trend. Another important factor that we take into account is volatility of observed markets compared to S&P 500. Standard volatility measurements would not fit into our approach so, we wanted to examine how much more will an index grow (fall) compared to change in S&P 500, if triggered by increase (decrease) in benchmark value.

4.3 Indices movements' unidirectional sensitivity

Results presented in Table 5 show average unidirectional sensitivity $\overline{\psi}$ calculated through mathematical formula (11) for unidirectional sensitivity (8). Only values with unidirectional correlation were taken into consideration.

Implication of results for average unidirectional sensitivity $\overline{\psi}$ is that it gives us a prediction of relative price movement for each regional index, as a result of previous day change in S&P 500, with a certain probability. Expected daily price movement for an index can be calculated as a multiple of S&P 500 relative change from previous trading day and ($\overline{\psi} + 1$).

Table 5. Average relative sensitivity

Indices pairs	Average Unidirectional Sensitivity Coefficients		
	$\overline{^{+}\psi^{xy}}$	$\overline{^{-}\psi^{xy}}$	$\overline{\psi^{xy}}$
BET - S&P 500	2.8431	2.7689	2.8078
BUX - S&P 500	2.9543	2.2547	2.6252
SOFIX - S&P 500	2.0544	2.1233	2.0878
BELEX LINE - S&P 500	2.7731	1.4164	2.0539
CROBEX - S&P 500	2.4398	1.4841	1.9899
SBI 20 - S&P 500	1.1617	1.2731	1.2132

This approach in data processing can be implemented in algorithmic trading system development. Note that in our research we used price relation of regional stock market indices to S&P 500, as a benchmark.

Discovery of other signals (i.e. single stock prices), with high linear correlation, can be basis for use of this method. In our work, we observed co-movements on a daily level but, model can be implemented on shorter time frames as well.

Comparing results for linear correlation and average unidirectional sensitivity, we can notice that there is a correspondence in ranking of linear correlation and unidirectional sensitivity (Table 6).

In other words, the higher a correlation is, the bigger is the price fluctuation compared to S&P 500.

Table 6. r_L and $\overline{\psi}$ comparison

Indices pairs	Comparison	
	r_L	$\overline{\psi}$
BET - S&P 500	0.9725	2.8078
BUX - S&P 500	0.9568	2.6252
SOFIX - S&P 500	0.9463	2.0878
BELEX LINE - S&P 500	0.9448	2.0539
CROBEX - S&P 500	0.9374	1.9899
SBI 20 - S&P 500	0.9011	1.2132

This is important result because it confirms theoretic claims that linear correlation and volatility coincide. We consider this result as a confirmation that average unidirectional sensitivity coefficient can be used as a volatility indicator.

5. CONCLUSIONS

In our paper, we analyzed relationship of price movements of six regional stock market indices and S&P 500, as a benchmark index. Due to different time zones, we compared previous trading day closing value of S&P 500 to each of six index current day closing values.

Examination of linear correlation coefficient r_L showed extremely high correlation values. Such a result confirmed high influence of S&P 500 on regional stock market's price movements.

In order to better understand regularity of price movement interdependence, we introduced unidirectional correlation.

Unidirectional correlation for each regional stock market describes how benchmark index S&P 500 and given market index simultaneously increase or decrease. Results show a probability that rise (fall) of S&P 500 will trigger rise (fall) of a regional index. This is important result for an emerging market investor because it can be used in decision making process.

Value of unidirectional correlation for an index contributes to investor's price prediction possibility and understanding of a market. Due to higher volatility of observed markets compared to S&P 500, we examined how much faster an index grows (falls) when triggered by increase (decrease) in benchmark value.

We introduced ψ, unidirectional sensitivity coefficient in order to describe quality of simultaneous movements of S&P 500 and each regional index. This coefficient, for a given index, can be implemented as a predictor of price percentage movement.

Multiplication of previous trading day S&P 500 change in value and unidirectional sensitivity coefficient $(\psi + 1)$, predicts index value with certain probability. Importance of unidirectional sensitivity coefficient is that it simplifies risk/reward analysis by adding quantitative perspective.

Our results also show that unidirectional sensitivity coefficient, for the given sample, has the same distribution with linear correlation. This result is a confirmation that average unidirectional sensitivity coefficient can be used as a volatility indicator.

Presented approach in quantification of unidirectional behavior of benchmark S&P 500 index and regional stock market indices can be applied in equity investing.

Implementation of approach should not be limited to stock market indices change prediction but, it should be examined in other cases where benchmark value can be established.

The unidirectional sensitivity research also has importance for development of algorithmic trading systems and it would be interesting to study it further on.

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6. APPENDIX 1: Theorems proofs

PROOF OF THEOREM 1.

If $\psi_i^{xy} \neq 0$ then $\rho_i^x - \rho_i^y \neq 0$, or $\rho_i^x \neq \rho_i^y$ and then we distinguish two events:

a) If $\rho_i^x > \rho_i^y$, then $\psi_i^{xy} \neq \psi_i^{yx}$.

b) If $\rho_i^x < \rho_i^y$, then $\psi_i^{xy} \neq \psi_i^{yx}$.

Based upon proven statements a) and b), initial theorem is consequently valid. Therefore, the proof is concluded.

PROOF OF THEOREM 2.

We will prove the theorem in three steps:

a) First, we will prove that $\psi_i^{x_1y} - \psi_i^{x_2y} > 0 \Rightarrow \delta_i^{x_1} - \delta_i^{x_2} > 0$.

Let us assume that $\psi_i^{x_1y} - \psi_i^{x_2y} > 0$, and then $\rho_i^{x_1} - \rho_i^{x_2} > 0$, from where follows $\delta_i^{x_1} - \delta_i^{x_2} > 0$.

b) Second, we will prove that $\psi_i^{x_1y} - \psi_i^{x_2y} = 0 \Rightarrow \delta_i^{x_1} - \delta_i^{x_2} = 0$.

Let us assume that $\psi_i^{x_1y} - \psi_i^{x_2y} = 0$, and then $\rho_i^{x_1} - \rho_i^{x_2} = 0$, from where follows $\delta_i^{x_1} - \delta_i^{x_2} = 0$.

c) Third, let us assume that $\psi_i^{x_1y} - \psi_i^{x_2y} < 0$, then $\rho_i^{x_1} - \rho_i^{x_2} < 0$, from where follows $\delta_i^{x_1} - \delta_i^{x_2} < 0$.

Based upon proven statements a), b) and c) follows that initial theorem is valid. Therefore, the proof is concluded.