International Journal of Industrial Engineering and Management (IJIEM), Vol.1 No 2, 2010, pp. 47-52
Available online at http://www.ftn.uns.ac.rs/ijiem
ISSN 2217-2661

Multiscale analysis of microstructure causality in the foreign exchange market

Nikola Gradojevic
Faculty of Business Administration, Lakehead University, 955 Oliver Road, Thunder Bay, ON P7B 5E1, Canada,
(University of Novi Sad, Faculty of Technical Sciences, Serbia)
ngradoje@lakeheadu.ca

Received 09. 09. 2010; Revised 27. 09. 2010; Accepted 25. 10. 2010

Abstract

Recent scholarly work on the causal relationship between exchange rate movements and currency order flows has provided mixed results. This paper proposes a wavelet approach for determining multiscale causality between the Canada/U.S. dollar returns and aggregate market order flow. Evidence of bi-directional causality that contradicts the microstructure literature is found for almost all time scales. Multiresolution analysis identifies significant structural breaks in the data that are potentially driving such findings.

Key words: *Wavelets; Causality; Foreign Exchange Markets; Market Microstructure*

1. INTRODUCTION

Foreign exchange (FX) market microstructure examines the elements of the currency trading process: the arrival and dissemination of information (via currency order flows) that is subsequently reflected in exchange rates and the FX market design which determines how orders are transformed into trades. Market order flow is a measure of *"excess demand"* that is defined as the difference between the currency purchases and sales for all relevant currency dealers over a period of time. Recent market microstructure literature has reported that currency order flows are powerful determinants and predictors of exchange rate returns (Evans and Lyons, 2002, 2005; Gradojevic, 2007; Osler and Vandrovych, 2009). The major assumption underlying both equity and FX market microstructure models is that price movements are driven by order flow. This argument can be found in the classical equity microstructure literature such as Hasbrouck (1991), Glosten and Milgrom (1985) and O'Hara (1995): in rational markets, order flow should reflect innovations in dispersed information, and not vice-versa.

Only a select few papers have directly tested the causality assumption in FX markets. Killeen et al. (2006) find that for the DM/FRF exchange rate Granger causality runs from interdealer order flow to price, and not vice versa. However, several papers documented statistically significant reverse causality effects. For instance, Sager and Taylor (2008) perform Granger causality tests on the data from Evans and Lyons (2002) and reveal that causality runs from the DM/USD and JPY/USD exchange rate returns to corresponding interdealer order flows. They also present evidence against the causality assumption for customer order flows. This evidence corroborates Marsh and O'Rourke (2005) who argue that commercial order flow is price sensitive. Similarly, Boyer and van Norden (2006) conclude that interdealer order flow responds to the FRF/USD spot rate innovations. They note that the price responsiveness of commercial order flow contrasts with the usual predictions of the microstructure literature. Gradojevic and Neely (2008) demonstrate the ability of the Canada/U.S. dollar returns to predict financial order flows, but not non-financial order flows. Finally, Lyons (2001) finds some evidence that falling prices induce additional selling in the JPY market and refers to that phenomenon as *"distressed selling".*

All of the above papers focus on testing the causality assumption at one particular data frequency (typically daily). In financial markets, the data generating process (DGP) is a complex network of layers with each layer corresponding to a particular frequency. A successful characterization of such DGP should be estimated with techniques that account for intraand inter-frequency dynamics (Dacorogna et al., 2001). By proposing a wavelet approach for determining multiscale causality, this paper provides a complete inter-frequency characterization of the DGP governing the causality relationship between aggregate market order flow and the Canada/U.S. dollar exchange rate returns. Specifically, such an approach investigates whether the existence as well as the direction of causality is frequency-dependent.[1]

[1]The idea that the causality relationship between two variables may have different characteristics at different time-scales can also be found in Gençay et al. (2001). They use wavelet multiresolution analysis of money growth and inflation, and show that for Argentina, Brazil, Chile, Israel, Mexico and Turkey the nature of the causality changes with wavelet scales (periods between two and 32 months).

These examinations offer a powerful new platform for examining the dynamics of causality between two variables, and complement and extend the current state of FX market microstructure literature. Moreover, the proposed multiscale (wavelet) framework can also be viewed as the one paving the future of economics, management and industrial engineering. Specifically, extensions of this approach such as wavelet variance decomposition, wavelet cross-covariance and wavelet cross-correlation can be used in any practical situation that involves nonstationary or transient time series. While the current work emphasizes the potential of wavelets, it is worthwhile to note that the future will inevitably bring refinements of this methodology and develop more sophisticated statistical methods for signal processing.

The results show very little evidence of a stable causal relationship between the market order flow and returns across all time scales. Moreover, the causality appears to be mostly bidirectional, which is in general inconsistent with the microstructure theory. Multiresolution analysis and decomposition of FX returns and order flow at multiple time scales identifies a significant structural break (or regime switch) in volatility of total order flow around 1999. Similar findings can be observed for various choices of wavelet filters up to the 7th scale, which corresponds to the 128-day (roughly 4-month) horizon.

In the next section, the wavelet methodology is explained. The data are briefly presented in Section 3. Section 4 discusses the findings and the final section concludes the paper.

2. WAVELET FRAMEWORK

Wavelet methods are rather newer ways of analyzing time series and can be seen as a natural extension of the Fourier analysis. The formal subject matter, in terms of their formal mathematical and statistical foundations go back only to the 1980s. In recent years, there have been several unique applications of wavelet methods to financial and econometric problems. Early applications of wavelets in economics and finance are by Ramsey and his coauthors (see Ramsey et al. (1995), Ramsey and Zhang (1997), Ramsey (1999), Ramsey (2002) for a review and references) who explore the use of discrete wavelet transformation (DWT) in decomposing various economic and financial data. Davidson et al. (1998) investigated U.S. commodity prices via wavelets. Gençay et al. (2003, 2005) propose a wavelet approach for estimating the systematic risk or the beta of an asset in a capital asset pricing model. The proposed method is based on a wavelet multiscaling approach where the wavelet variance of the market return and the wavelet covariance between the market return and a portfolio are calculated to obtain an estimate of the portfolio's systematic risk (beta) at each scale. In time series econometrics, one example of the successful application of wavelets is in the context of long memory processes where a number of estimation methods have been developed. These include wavelet-based OLS, the approximate wavelet-based maximum likelihood approach, and wavelet-based Bayesian approach. Fan (2003) and Fan and Whitcher (2003) provide an extensive list of references. The success of these methods relies on the so called *"approximate decorrelation"* property of the DWT of a possibly nonstationary long memory process.[2] Fan and Whitcher (2003) propose overcoming the problem of spurious regression between fractionally differenced processes by applying the DWT to both processes and then estimating the regression in the wavelet domain. Other examples of applications of wavelets in econometrics include wavelet-based spectral density estimators and their applications in testing for serial correlation/conditional heteroscedasticity, see e.g., Hong (2000), Hong and Lee (2001), Lee and Hong (2001), Duchesne (2006a), Duchesne (2006b), and Hong and Kao (2004). Gençay and Fan (2009) that applied wavelets to test the presence of a unit root in a stochastic process. By using Monte Carlo simulations, they demonstrated the comparable power of the wavelet-based tests with reasonable empirical sizes.

A wavelet is a small wave which grows and decays in a limited time period.[3] To formalize the notion of a wavelet, let $\psi(.)$ be a real valued function such that its integral zero,

$$\int_{-\infty}^{\infty} \psi(t)\, dt = 0, \tag{1}$$

and its square integrates to unity,

$$\int_{-\infty}^{\infty} \psi(t)^2\, dt = 1. \tag{2}$$

Wavelets are, in particular, useful for the study of how weighted averages vary from one averaging period to the next. Let $x(t)$ be real-valued and consider the integral

$$\bar{x}(s,e) \equiv \frac{1}{s-e}\int_s^e x(u)\, du \tag{3}$$

where we assume that $e > s$. $\bar{x}(s, e)$ is the average value of $x(.)$ over the interval $[s, e]$. Instead of treating an average value $\bar{x}(s, e)$ as a function of end points of the interval $[s, e]$, it can be considered as a function of the length of the interval,

$$\lambda \equiv s - e$$

while centering the interval at

$$t = (s+e)/2.$$

λ is referred to as the scale associated with the average, and using λ and t, the average can be redefined such that

$$a(\lambda, t) \equiv \bar{x}(t - \frac{\lambda}{2}, t + \frac{\lambda}{2}) = \frac{1}{\lambda}\int_{t-\frac{\lambda}{2}}^{t+\frac{\lambda}{2}} x(u)\, du$$

[2] See Fan (2003) for a rigorous proof of this result for a nonstationary fractionally differenced process.

[3] The contrasting notion is a big wave such as the sine function which keeps oscillating indefinitely.

where $a(\lambda, t)$ is the average value of $x(.)$ over a scale of λ centered at time t. The change in $a(\lambda, t)$ from one time period to another is measured by

$$w(\lambda, t) \equiv a(\lambda, t+\frac{\lambda}{2}) - a(\lambda, t-\frac{\lambda}{2}) = \frac{1}{\lambda}\int_{t}^{t+\lambda} x(u)\,du - \frac{1}{\lambda}\int_{t-\lambda}^{t} x(u)\,du. \quad (4)$$

Equation 4 measures how much the average changes between two adjacent nonoverlapping time intervals, from $t - \lambda$ to $t + \lambda$, each with a length of λ. Because the two integrals in Equation 4 involve adjacent nonoverlapping intervals, they can be combined into a single integral over the real axis to obtain

$$w(\lambda, t) = \int_{-\infty}^{\infty} \tilde{\psi}(t) x(u)\,du \quad (5)$$

where

$$\tilde{\psi}(t) = \begin{cases} -1/\lambda, & t-\lambda < u < t, \\ 1/\lambda, & t < u < t+\lambda, \\ 0, & \text{otherwise.} \end{cases}$$

$\omega(\lambda,t)$'s are the wavelet coefficients and they are essentially the changes in averages across adjacent (weighted) averages.

2.1 Discrete wavelet transformation

In principle, wavelet analysis can be carried out in all arbitrary time scales. This may not be necessary if only key features of the data are in question, and if so, DWT is an efficient and parsimonious route as compared to the continuous wavelet transformation (CWT). The DWT is a subsampling of $\omega(\lambda,t)$ with only dyadic scales, i.e., λ is of the form 2^{j-1}, j = 1, 2, 3, ... and, within a given dyadic scale 2^{j-1}, t's are separated by multiples of 2^j .

Let x be a dyadic length vector ($N = 2^J$) of observations. The length N vector of discrete wavelet coefficients w is obtained by

$$\mathbf{w} = \mathcal{W}\mathbf{x},$$

where W is an $N \times N$ real-valued orthonormal matrix (based on the wavelet type) defining the DWT which satisfies $W^T W = IN$ ($n \times n$ identity matrix).[4] The nth wavelet coefficient ω_n is associated with a particular scale and with a particular set of times. The vector of wavelet coefficients may be organized into $J + 1$ vectors,

$$\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_J, \mathbf{v}_J]^T,$$

where w_j is a length $N/2^j$ vector of wavelet coefficients associated with changes on a scale of length $\lambda_j = 2^{j-1}$ and v_J is a length $N/2^J$ vector of scaling coefficients associated with averages on a scale of length $2^J = 2\lambda_J$.

[4]Since DWT is an orthonormal transform, orthonormality implies that $x = W^T w$ and $||w||^2 = ||x||^2$.

Using the DWT, we may formulate an additive decomposition of x by reconstructing the wavelet coefficients at each scale independently.

Let $d_j = W_j^T w_j$ define the jth level *wavelet detail* associated with changes in x at the scale λ_j (for $j = 1,\ldots, J$).

The wavelet coefficients $w_j = W_j x$ represent the portion of the wavelet analysis (decomposition) attributable to scale λ_j, while $W_j^T w_j$ is the portion of the wavelet synthesis (reconstruction) attributable to scale λ_j.

For a length $N = 2^J$ vector of observations, the vector d_{J+1} is equal to the sample mean of the observations.

A multiresolution analysis (MRA) may now be defined via

$$x_t = \sum_{j=1}^{J+1} \mathbf{d}_{j,t} \quad t = 1, \ldots, N \quad (6)$$

That is, each observation x_t is a linear combination of wavelet detail coefficients at time t. Let $s_j = P_{k=j+1}^{J+1} d_k$ define the jth level wavelet smooth.

Whereas the wavelet detail d_j is associated with variations at a particular scale, s_j is a cumulative sum of these variations and will be smoother and smoother as j increases. In fact, $x - s_j = P_{k=1}^{J}\ d_k$ so that only lower-scale details (high-frequency features) from the original series remain.

The jth level *wavelet rough* characterizes the remaining lower-scale details through

$$\mathbf{r}_j = \sum_{k=1}^{j} \mathbf{d}_k, \quad 1 \le j \le J+1$$

The wavelet rough rj is what remains after removing the wavelet smooth from the vector of observations. A vector of observations may thus be decomposed through a wavelet smooth and rough via

$$x = s_j + r_j\ ,$$

for all j.

The terminology *"detail"* and *"smooth"* were used by Percival and Walden (2000) to describe additive decompositions from Fourier and wavelet transforms. The goal is to look at data at different resolutions with this representation. The smooth part is coarse: we are looking at local averages, i.e., low-frequency trends and the sample mean. The detail is deviation from the smooth part.

A variation of the DWT is called the maximum overlap DWT (MODWT). Similar to the DWT, the MODWT is a subsampling at dyadic scales, but in contrast to the DWT, the analysis involves all times t rather than the multiples of 2^j.

Retainment of all possible times eliminates alignment effects of DWT and leads to more efficient time series representation at multiple time scales.

3. DATA DESCRIPTION

The data is at a daily frequency and are obtained from the Bank of Canada. The data spans the period between January 2, 1990 and December 31, 2004. If S_t denotes an exchange rate at time t, then exchange rate returns are defined as $r_t = log(S_t) - log(S_{t-1})$.

The order flow data were obtained from the Bank of Canada and they are four types of daily trading flows (in Canadian dollars) for eight major Canadian commercial banks:

- Commercial client transactions (CC) include all transactions with resident and nonresident non-financial customers (transactions with banks, investment dealers, or other non-bank financial institutions such as trusts, life insurance, and investment funds are excluded);
- Foreign institution transactions (FD) include all transactions with foreign banks, branches and subsidiaries of Canadian banks located outside Canada, foreign investment dealers and foreign non-bank financial institutions;
- Canadian-domiciled investment transactions (CD) include all transactions with nondealer financial institutions located in Canada;
- Interbank transactions (IB) pertain to other Canadian-domiciled financial institutions, such as chartered banks, credit unions, investment dealers, and trust companies.

These order flows represent approximately 40-60% of all Canada/U.S. dollar transactions and their arithmetic sum is the daily aggregate market order flow ($x_t = CC_t+FD_t+CD_t+IB_t$). Using the definition from Lyons (2001), order flows are measured as the difference between the number of currency purchases (buyer-initiated trades) and sales (seller-initiated trades). Ceteris paribus, positive (negative) order flow should raise (lower) the Canada/U.S. dollar spot closing rates (S_t), appreciating (depreciating) the USD. Modified Phillips-Perron test, the Elliott-Rothenberg-Stock test and the augmented Dickey-Fuller test reject the null hypothesis of a unit root in both the FX returns and the total order flow series.

4. RESULTS

To assess causality at different time scales, Granger causality tests are applied on wavelet details of FX returns (r_t) and total order flow (x_t) series. The Wald test is used to test two null hypotheses: 1) order flow does not Granger cause returns and 2) returns do not Granger cause order flow. First, the MODWT decomposes the original time series into different time scales using the S(8) filter. As the data are daily, the first scale is defined roughly for the 2 day horizon, the second scale for 4 days, etc. Table 1 shows the results of causality tests for the first seven scales. It can be seen that bi-directional causality is present at each time scale up to 128 days (4 months). Thus, even in long run, reverse causality effects are dominant which contrasts the microstructure literature. This suggests that regression estimates based on this data set would be biased.

Table 1. Granger causality between returns and total order flow using S(8) filter

	S(8)
j=1	$r_t \leftrightarrow_{1\%} x_t$
j=2	$r_t \leftrightarrow_{1\%} x_t$
j=3	$r_t \leftrightarrow_{1\%} x_t$
j=4	$r_t \leftrightarrow_{1\%} x_t$
j=5	$r_t \leftrightarrow_{1\%} x_t$
j=6	$r_t \leftrightarrow_{1\%} x_t$
j=7	$r_t \leftrightarrow_{1\%} x_t$

The null hypotheses are: 1) order flow does not Granger cause returns ($r_t \leftarrow x_t$) and 2) returns do not Granger cause order flow ($r_t \leftarrow x_t$). Arrows denote the direction of Granger causality between FX returns (r_t) and aggregate market order flow (x_t) that is statistically significant at the 1% significance level.

To investigate the robustness of the results to the choice of wavelet filters, granger causality test results are also reported for the MODWT using S(4), D(4) and Haar filters (Table 2).

Table 2. Granger causality between returns and total order flow using S(4), D(4) and Haar filters

	S(4)	D(4)	Haar
j=1	$r_t \leftrightarrow_{1\%} x_t$	$r_t \leftrightarrow_{1\%} x_t$	$r_t \leftrightarrow_{1\%} x_t$
j=2	$r_t \leftrightarrow_{1\%} x_t$	$r_t \leftrightarrow_{1\%} x_t$	$r_t \leftrightarrow_{1\%} x_t$
j=3	$r_t \leftrightarrow_{1\%} x_t$	$r_t \leftrightarrow_{1\%} x_t$	$r_t \leftarrow_{1\%} x_t$
j=4	$r_t \leftrightarrow_{1\%} x_t$	$r_t \leftrightarrow_{1\%} x_t$	$r_t \leftrightarrow_{1\%} x_t$

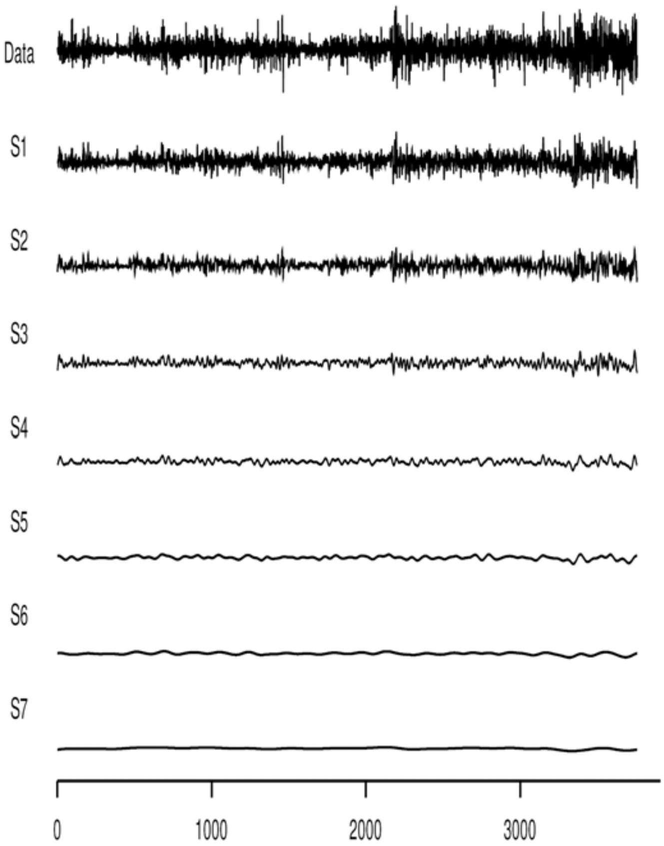
The null hypotheses are: 1) order flow does not Granger cause returns (rt xt) and 2) returns do not Granger cause order flow (rt ! xt). Arrows denote the direction of Granger causality between FX returns (rt) and aggregate market order flow (xt) that is statistically significant at the 1% significance level.

The results are in general similar to Table 1, except for the Haar filter when order flow granger causes returns at the third time scale (i.e., weekly data horizon), but reverse causality is not found.[5]

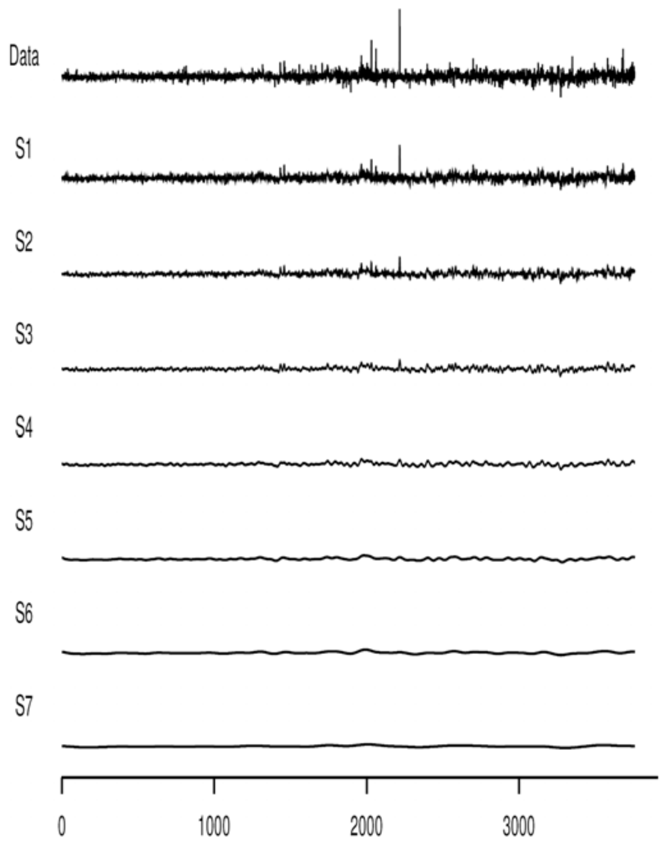
Next, multiresolution analysis (MRA) and decomposition of returns and aggregate order flow at multiple time scales are conducted. Figure 1 provides multiresolution analysis and Figure 2 presents multiresolution decomposition (MRD). The difference between multiresolution analysis and decomposition is that the former examines the smooth and the latter examines the detail component of the raw data series.

From Figure 1 it appears that there exist sharp increases and decreases in the variability of both FX returns and order flow series. Specifically, for FX returns, candidates for regime shifts are periods around 1994-1995 when the volatility decreased and 1999 when an increase in volatility was permanent and lasted until the last day of the sample. These fluctuations are followed by the order flow behavior around 1999 when it exhibits a significant spike. Such effects are particularly pronounced for scales 1 to 4. Multiresolution decomposition (Figure 2) confirms these findings and pins down the period around 1999 as the major structural break in the data.

[5] The results for wavelet smooth are consistent with the results for wavelet detail and they can be available upon request from the author.

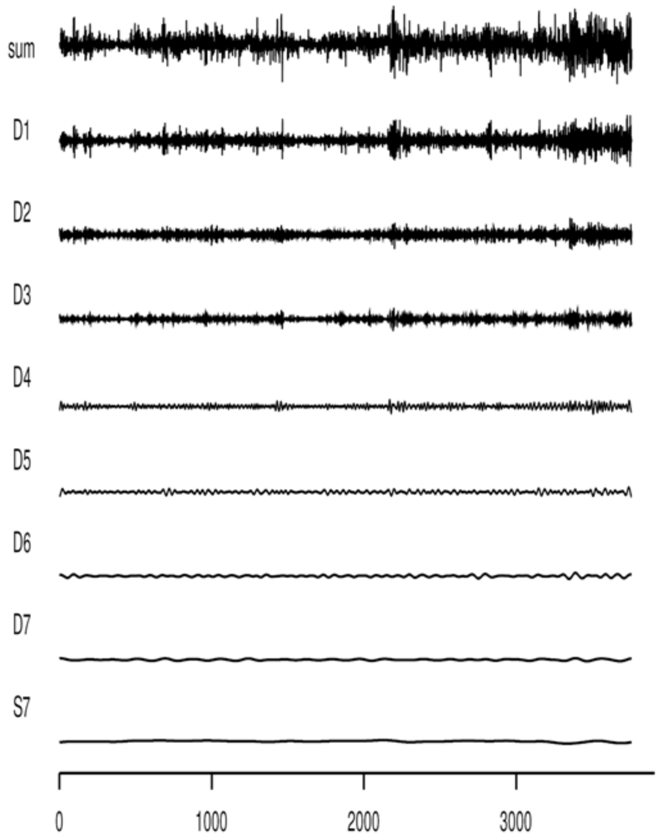


(a) MRA of r_t

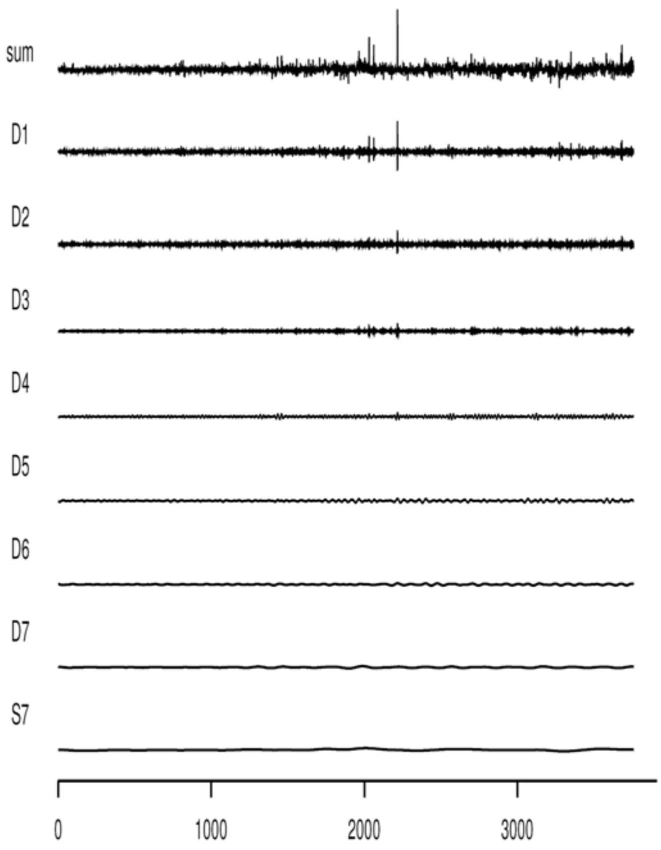


(b) MRA of x_t

Figure 1. Multiresolution analysis of returns and total order flow. The multiresolution analysis is conducted using S(8) filter. The left panel shows the smooth component of FX returns at scales from 1 to 8. The right panel shows the smooth component of total order flow at scales from 1 to 8



(a) MRD of r_t



(b) MRD of x_t

Figure 2. Multiresolution decomposition of returns and total order flow. The multiresolution decomposition is conducted using S(8) filter. The left panel shows the detail component of FX returns at scales from 1 to 8. The right panel shows the detail component of total order flow at scales from 1 to 8

5. CONCLUSIONS

The goal of this study is to critically investigate the causality relationship between spot exchange rates and aggregate currency order flow in the Canada/U.S. dollar market. In particular, by utilizing the wavelet methodology, the paper tests the theoretical prediction of market microstructure literature that order flow drives exchange rates (and not vice versa).

The results offer little reason for optimism: Granger causality is found to persistently run both ways, from order flow to FX returns and in the reverse direction. The presence of a perverse causal relationship at all time scales could arise from a number of factors. First, it may simply be that the nature of order flow-exchange rate interactions is such that it requires dynamic models such as a VAR or VECM system where both variables would be determined endogenously. Second, the Bank of Canada data may be too noisy and unrepresentative of the total Canada/U.S. dollar order flow, i.e., subject to the errors-in-variables problem. Third, reverse causality may reflect the predominance of technical trading strategies at all horizons.

Finally, given the evidence of structural breaks in 1998-1999, it may be that the relationship between order flow and returns is inherently unstable. Noteworthy, the 1998-1999 period cover the August 1998 Russian default and the September 1998 collapse of Long-Term Capital Management, while the Canadian dollar depreciated by 10 cents in August 1998.

6. REFERENCES

[1] Berger, D., Chaboud, A., Chernenko, S., Howorka, E., and Wright, J. (2008) *"Order flow and exchange rate dynamics in electronic brokerage system data"*, Journal of International Economics, 75(1), 93-109.

[2] Boyer, M. and van Norden, S. (2006) *"Exchange rates and order flow in the long run"*, Finance Research Letters, 3, 235243.

[3] Breitung, J. and Candelon, B. (2006) *"Testing for short- and long-run causality: A frequencydomain approach"*, Journal of Econometrics, 132, 363378.

[4] Dacorogna, M., Gençay, R., Muller, U., Olsen, R., and Pictet, O. (2001) *"An Introduction to High-Frequency Finance"*, Academic Press, San Diego.

[5] Davidson, R., Labys, W. C., and Lesourd, J.-B. (1998) *"Walvelet analysis of commodity price behavior"*, Computational Economics, 11, 103-128.

[6] Duchesne, P. (2006a). *"On testing for serial correlation with a wavelet-based spectral density estimator in multivariate time series"*, Econometric Theory, 22, 633-676.

[7] Duchesne, P. (2006b) *"Testing for multivariate autoregressive conditional heteroskedasticity using wavelets"*, Computational Statistics & Data Analysis, 51, 2142-2163.

[8] Evans, M. and Lyons, R. (2002) *"Order flow and exchange rate dynamics"*, Journal of Political Economy, 110, 170-180.

[9] Evans, M. and Lyons, R. (2005) *"Meese-Rogoff redux: Micro-based exchange rate forecasting"*, American Economic Review, 95(2), 405–414.

[10] Fan, Y. (2003) *"On the approximate decorrelation property of the discrete wavelet transform for fractionally differenced processes"*, IEEE Transactions on Information Theory, 49, 516-521.

[11] Fan, Y. and Whitcher, B. (2003) *"A wavelet solution to the spurious regression of fractionally differenced processes"*, Applied Stochastic Models in Business and Industry, 19, 171-183.

[12] Gençay, R. and Fan, Y. (2009) *"Unit root tests with wavelets. Econometric Theory"*, Forthcoming.

[13] Gençay, R., Sel¸cuk, F., and Whitcher, B. (2001) *"An Introduction to Wavelets and Other Filtering Methods in Finance and Economics"*, Academic Press, San Diego.

[14] Gençay, R., Sel¸cuk, F., and Whitcher, B. (2003) *"Systematic risk and time scales"*, Quantitative Finance, 3, 108-116.

[15] Gençay, R., Sel¸cuk, F., and Whitcher, B. (2005) *"Multiscale systematic risk"*, Journal of International Money and Finance, 24, 55–70.

[16] Glosten, L. and Milgrom, P. (1985) *"Bid, ask and transaction prices in a specialist market with heterogeneously informed traders"*, Journal of Financial Economics, 14, 71–100.

[17] Gradojevic, N. (2007). "The microstructure of the Canada/U.S. dollar exchange rate: A robustness test". Economics Letters, 94(3), 426-432.

[18] Gradojevic, N. and Neely, C. (2008) *"The dynamic interaction of trading flows, macroeconomic announcements and the CAD/USD exchange rate: Evidence from disaggregated data"*, Working Paper 2008-006C.

[19] Hasbrouck, J. (1991) *"Measuring the information content of stock trades"*, Journal of Finance, 46, 179207.

[20] Hong, Y. (2000) *"Wavelet-based estimation for heteroskedasticity and autocorrelation consistent variance-covariance matrices"*, Technical report, Department of Economics and Department of Statistical Science, Cornell University.

[21] Hong, Y. and Kao, C. (2004) *"Wavelet-based testing for serial correlation of unknown form in panel models"*, Econometrica, 72, 1519-1563.

[22] Hong, Y. and Lee, J. (2001) *"One-sided testing for ARCH effects using wavelets"*, Econometric Theory, 17, 1051-1081.

[23] Killeen, W., Lyons, R., and Moore, M. (2006) *"Fixed versus flexible: Lessons from EMS order flow"*, Journal of International Money and Finance, 25(4), 551-579.

[24] Lee, J. and Hong, Y. (2001) *"Testing for serial correlation of unknown form using wavelet methods"*, Econometric Theory, 17, 386–423.

[25] Lyons, R. (2001) *"The Microstructure Approach to Exchange Rates"*, The MIT Press, Cambridge, USA.

[26] Marsh, I. W. and O'Rourke, C. (2005) *"Customer order flow and exchange rate movements: Is there really information content?"*, Working Paper.

[27] O'Hara, M. (1995) *"Market Microstructure Theory"*, Blackwell Publishers, Cambridge, Mass. Osler, C. and Vandrovych, V. (2009). Hedge funds and the origins of private information in currency markets. Working Paper No. 1484711.

[28] Percival, D. B. and Walden, A. T. (2000) *"Wavelet Methods for Time Series Analysis"*, Cambridge Press, Cambridge.

[29] Ramsey, J. B. (1999) *"The contribution of wavelets to the anlaysis of economic and financial data"*, Philosophical Transactions of the Royal Society of London A, 357, 2593-2606.

[30] Ramsey, J. B. (2002) *"Wavelets in economics and finance: Past and future"*, Studies in Nonlinear Dynamics & Econometrics, 3, 1–29.

[31] Ramsey, J. B. and Zhang, Z. (1997) *"The analysis of foreign exchange data using waveform dictionaries"*, Journal of Empirical Finance, 4, 341-372.

[32] Ramsey, J. B., Zaslavsky, G., and Usikov, D. (1995) *"An analysis of U. S. stock price behavior using wavelets"*, Fractals, 3, 377-389.

[33] Sager, M. and Taylor, M. (2008) *"Commercially available order flow data and exchange rate movements: Caveat emptor"*, Journal of Money, Credit and Banking, 40(4), 583625.